ECE 250 *Algorithms and Data Structure*
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with the subject ECE 250 Questions 9.2.
Assistances and comments will be acknowledged.

9.2*a* Suppose you have a complex number class `template <typename Type> complex<Type>`. Why would it not be appropriate to use the address of the complex number as a hash function?

9.2*b* Suppose you create a new object storing information about a student and where the hash function will only be used for operations to access and save data to the object. Could you use the address of the object for a hash function?

9.2*c* Suppose you require the hash value of an object to be persistent from one session to another where the object may be stored to memory in the interim and reloaded. Why would using the address not be appropriate in this case?

9.2*d* One way for generating a 32-bit hash value of a string is to take the first four bytes and treat them as an integer:

```
unsigned int hash( const string &str ) {
    unsigned int hash_value = 0;

    for ( int k = 0; k < str.length(); ++k ) {
        hash_value += str[k] << (24 – 8*(k && 3));
    }

    return hash_value;
}
```

Thus, `"hello"`, which is stored as

```
01001000  01100101  01101100  01101100  01101111  00000000
```

would have the hash value $01001000011001010110110001101100_2 = 1214606444$.

9.2*e* Suppose that there are 120 students in your class and 365.25 days per year. Thus, we should have approximately $\lambda = 0.328542$ students born each day. If we calculate $\frac{\lambda^n}{n!}e^{-\lambda}$ for n = 0, 1, 2, 3, 4, and 5, we get the values

n	$\frac{\lambda^n}{n!}e^{-\lambda}$
0	0.71997262
1	0.23654131
2	0.038856889
3	0.0042553746
4	0.00034951742
5	0.000022966237

Thus, we expect 0.038856889 or 3.8856889 % of days to have two birthdays appear on that day. Thus, approximately fourteen pairs of students should share birthdays in any class of size 120. How many triplets of students should share three birthdays?

9.2*f* Continuing from Question 9.2*e*, suppose we have the same class with 120 students and use the approximation that there are twelve equally long months each year. What is the number of months that should have 10 students born in that month? What is the probability that a month will have no students born in that month?

9.2*g* Show that

$$\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} = 1$$

for all values of λ; whether positive or negative, real or complex. You may wish to recall the formula for the Taylor-series expansion of the exponential function to do this question.

9.2*h* Suppose that it is absolutely necessary that a hash value of a string changes with any change to the string itself (adding, changing, or removing a character). What is the optimal run time of such an algorithm?

Follow up: cryptographic hash functions are hash functions where any accidental or deliberate change in the original text results in a different hash value. As these hash functions are summaries, of the original text, they are also called *message digests*. Look up SHA-2 (Secure Hash Algorithm).

9.2*i* A double-precision floating-point number has a 64-bit number. Come up with a hash function that produces a 32-bit number. Note that +0 and –0 should have the same hash value. These have hexadecimal representations of `0x0000000000000000` and `0x8000000000000000`, respectively.