What is the Right Model for Wireless Channel Interference?

Aravind Iyer, Catherine Rosenberg and Aditya Karnik

Abstract— **In a wireline network, nodes form links with only those nodes they are *wired* to, and the links do not interfere with one another. In contrast, in a wireless network, signal transmissions are intrinsically *broadcast*, and suffer from mutual interference. In several physical layer technologies, a wireless signal is decoded by treating *the sum* of all the other on-going signal transmissions as noise. Hence, from a networking standpoint, there is *a need to model* wireless channel interference. An accurate interference model is especially important in a multi-hop network context, since there could be several simultaneous wireless transmissions. Several works in the literature have made use of simplified interference models. Some works assume a fixed range for communication and interference, while others are based on concepts like capture threshold where the desired signal strength is compared with interference from a *single* node at a time, rather than *cumulatively*. In particular, the latter model is used in *ns2* which is the most common simulation tool. Under isotropic pathloss, the capture threshold model is also equivalent to the protocol model proposed by Gupta and Kumar, which is now the subject of a lot of analytical activity notably through conflict graph based problem formulations. We investigate the accuracy and appropriateness of the capture threshold based interference model, by comparing it with one based on the SINR (signal-to-interference-and-noise ratio) with *additive* interference calculation. We find that both in the case of random access networks, as well as in the case of scheduled networks (where node transmissions are scheduled to be completely conflict-free), a simplified interference model such as the capture threshold model, can produce significantly different results compared to an additive interference based model. Therefore, a lot of *caution* should be exercised before accepting or interpreting results based on simplified interference models.**

I. Introduction

Wireless communication networks have seen a phenomenal amount of growth in the last few years. Cellular networks have expanded their penetration tremendously, while infrastructure-based wireless local area networks (WLANs) have proliferated beyond campuses and airports, to provide almost ubiquitous wireless connectivity in several urban areas. Interest in wireless networks is expected to continue growing, as networks such as wireless sensor networks, ad hoc networks, and wireless mesh networks begin to see deployment. Two key common denominators of existing and upcoming wireless networking applications, are (i) limited transmit power, and (ii) the scarcity of wireless channel bandwidth. This makes the design and optimization of wireless networks critical. Therefore, it is important to carefully understand, from a networking standpoint, how wireless communications are different from wireline.

In a wireline network, nodes form links with only those nodes they are *wired* to. These wired links can be abstracted using a few simple (non time-varying) parameters such as propagation delay, and link capacity, and the operation of each link is unaffected by the others. In contrast, in a wireless network, signal transmissions are intrinsically *broadcast*. Specifically, the wireless medium has the following three features. Firstly, the signal power from a wireless transmitter is radiated into space, rather than being confined within a wire. Secondly, wireless signals get attenuated considerably over distance, due to a combination of pathloss, location-dependent shadowing, and time-varying fading. Thirdly, in several physical layer technologies, a wireless signal is decoded by treating *the sum* of all the other on-going signal transmissions as noise. The decoding is probabilistic and the signal-to-interference-and-noise ratio (SINR) has to exceed a threshold to guarantee packet reception at a certain (bit/packet) error rate.

Clearly, unlike a wireline medium, the wireless physical channel cannot be abstracted using simple per-link parameters. In particular, from a networking standpoint, there is *a need to model* wireless

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channel interference, in order to characterize the simultaneous operation of a set of wireless links. An accurate model for interference, is certainly important from the analytical standpoint, so as to get the right insights in designing and optimizing wireless networks. From a simulation standpoint, the importance of a realistic model cannot be overemphasized, since simulations are used extensively to evaluate the performance of newly proposed protocols and algorithms. The aim of this paper therefore, is to ask the question: *what is the right model for wireless channel interference?* Specifically, we would like to investigate what is the right model for interference in multi-access networks such as ad hoc networks, sensor networks and mesh networks, where communication is based on the time-sharing of a single channel, either via random access or scheduling. Towards this end, let us begin by briefly tracing the evolution of various existing models for wireless channel interference.

One of the simplest models for the wireless channel, is based on the assumption of nodes being *co-located*, *à la* Bianchi [1]. In other words, all the nodes are assumed to be able to communicate with, and to cause interference to one another. This models the shared nature of the wireless medium, and can be considered a reasonable model for single-hop networks, but it does not model the existence of *hidden* nodes[1]. In order to model the existence of hidden (and exposed and deaf) nodes, models based on the notion of fixed communication and interference ranges have been considered (for instance, in [12], [14], [19]). Thereby, by definition, nodes within *communication range* can communicate with one another. Further for a given active transmitter-receiver pair, any node within the *interference range* of the receiver (again, by definition) can cause a packet collision at the receiver by transmitting.

In reality however, collisions do not occur merely due to concurrency of signal transmissions within some range, but only depending on the relative signal strength of the packet being received. The capture threshold model adopted in the network simulator *ns2* [15], tries to model the phenomenon of *capture*. Rather than use a fixed interference range, or assume nodes to be co-located, collisions are determined by comparing the desired signal strength at the receiver, with the level of interference. However, the signal strength of the packet being received, is compared with the interfering power from a single node at a time, rather than using the SINR with cumulative interference, to determine successful reception. In the paper on the capacity of wireless networks [4], Gupta and Kumar used what they termed the *protocol model* of interference. Under the protocol model, nodes within some range can communicate, provided no active interferer exists within a distance of the receiver that is proportional to the transmitter-receiver separation. As we shall see later, under isotropic pathloss, this is equivalent to the capture threshold model of interference.

To summarize, even though models for wireless channel interference have evolved, they still do not accurately model the three characteristics of the wireless medium, identified earlier. Indeed, to the best of our knowledge these models have been extensively used in the literature, without validation. In particular, inspite of its simplistic wireless channel model, the network simulator *ns2* continues to finds widespread use to evaluate new proposals for routing and medium access control (MAC) protocols. Further, an interference model such as the protocol model of [4], leads to *binary* conflict relationships among the wireless links (*i.e.*, any two links either interfere with each other, or they can be active simultaneously, regardless of the other on-going signal transmissions which is clearly not true in practice). The protocol model, and the related *binary* interference models, are currently the subject of a lot of analytical activity (for example, see [2], [8], [11]). While these works are important as stepping stones, their validity needs to be examined carefully before putting them to use.

This paper seeks to address this issue. Our aim is to understand which is the right model for wireless channel interference, and how accurate are existing simplified models, in particular, the capture threshold model. For this, we compare the capture threshold model of interference with one based on SINR thresholding to model capture, with additive (rather than single) interference calculation. Note that our main focus is not on modeling *signal propagation* on the wireless channel, but rather on modeling *interference*. Specifically, we would like to investigate the shortcomings of determining packet collisions using the interference from single

[1]With respect to a transmitter-receiver pair, a node or group of nodes is said to be *hidden*, if they cannot perceive any wireless signal from the transmitter, but they can cause a packet reception failure (*i.e.*, a *collision*) at the receiver if they themselves transmit.

nodes, one at a time, rather than cumulatively.

We consider two case-studies. First we use a discrete-event simulator to simulate different MAC protocols under both channel models. We find that not only does the capture threshold model overestimate the throughput performance in all cases, but it often predicts different qualitative behavior. Second, we consider an optimal routing problem (which we formulate and explore in much greater detail, in a companion paper [7]) to maximize the throughput of a scheduled network (where node transmissions are scheduled to be free of collisions). We get some numerical results for the two models. We find that again the capture threshold model overestimates the optimal throughput. Further, the optimal routing predicted by the two models can be quite different. Our conclusions are twofold. Firstly, we would like to advocate exercising caution before accepting results based on simplified models. Secondly, we feel that an additive interference model is the minimum level of detail that should be modeled, in order to obtain accurate results and insights.

In the remainder of the paper, we start with a survey of related work in Section II, followed by an introduction to the wireless channel interference models that we investigate in this paper in Section III. In Section IV, we compare the performance of a number of MAC protocols using a discrete-event simulator, under both the capture threshold and the additive interference model. In Section V, we consider an optimal routing problem on a scheduled network, and get some numerical results to illustrate the difference between the two channel models. Finally, we conclude our paper in Section VI.

II. Related Work

There has been a lot of research activity in the area of wireless communication networks, over the past decade or so. Much work has been devoted to characterizing the throughput capacity and optimal operation of a multi-hop wireless network, and also on developing algorithms, and protocols to enable the same. However, the study of simulation models, and common assumptions, and their experimental validation, has received relatively less attention. Notably, [17], [18] have investigated the impact of wireless physical channel modeling, on simulations and the performance of MAC and routing protocols, while in [10], the authors have tried to experimentally test commonly held assumptions about the wireless medium.

In [17], Takai *et. al.* investigate different models of wireless signal propagation, and the computational effect of increasingly sophisticated models, on large scale simulations of wireless networks. They argue that modeling wireless signal propagation merely as pathloss attenuation is not completely realistic. Although detailed models of signal propagation, can slow down execution times for wireless simulations, techniques such as parallel computation can help reduce the cost of computation. While we are also interested in wireless channel modeling, our work is different from [17] in that our main focus is not on modeling the wireless signal propagation, but on modeling interference. Specifically, we show that if packet collisions are determined using the interference from single nodes, one at a time, instead of using the cumulative interference and noise, then the conclusions derived could be misleading.

In a closely related work [18], the authors investigate the effects of physical channel modeling on the performance of MAC and routing protocols. The authors compare the performance of the IEEE 802.11 MAC protocol [5] using two simulators, GloMoSim [3] and *ns2* [15]. The authors rightly point out that since *ns2* uses a simplistic capture threshold model for inferring collision or capture, it overestimates the throughput performance of IEEE 802.11. In addition, the paper also compares the routing protocols DSR [9] and AODV [16], under different models of wireless signal propagation, and finds that one can arrive at different conclusions as to which routing protocol is better, depending on the model. While our work supports the findings in [18], we further show that not only is the capture threshold model optimistic as compared to an additive interference model, but there are qualitative differences in the predictions of the two models, and they can result in different structural insights.

In [10], the authors take an experimental approach to evaluate the validity of the common assumptions made in papers on wireless networks. The authors claim that several simplifying assumptions such as fixed transmission ranges, symmetric links, identical ranges for all radios, and so on, are still extant among the papers being published. Through extensive experimentation, the authors provide some recommendations for a good probabilistic signal propagation model. While such models can be developed, and incorporated in simulators (such as *ns2*, for instance), we show that it is also important to have an accurate model for interference calculation: specifically that the cumulative interference and noise should be used to determine capture.

As mentioned earlier, in works like [12], [14], [19], the authors assume a fixed communication range and a fixed interference range. In [20], the authors assume the capture threshold model of interference, and use *ns2* to evaluate their proposed MAC protocol, entitled DUCHA. Other works like [2], [8], [11] make use of the protocol model or a related *binary* interference model, although the authors of [8] do extend their work to an additive SINR based model.

III. Models for Wireless Channel Interference

In this section, we introduce the models for wireless channel interference that we wish to investigate. The physical channel model we advocate for use, is based on additive interference, and SINR thresholding for correct packet reception. This is motivated by the decoding strategies employed in several wireless physical layer technologies. Thereby, a wireless signal is decoded by treating the sum of all the other on-going signal transmissions and environmental disturbances, as noise. The decoding is probabilistic, and the success or failure of a signal transmission can be expressed in terms of a bit/packet error probability which depends on the SINR[2]. For a certain acceptable bit error rate (BER), the SINR has to exceed an appropriate threshold. Hence, a sufficient condition for packet reception is that the SINR remains greater than the threshold, throughout the duration of the packet transmission.

A. Additive Interference Model

Formally, denoting the set of simultaneously active wireless links (or transmitter-receiver pairs) by $\mathcal{L}$, and the transmit power and location of the transmitter of link $l \in \mathcal{L}$, as P_l and X_l respectively, the SINR perceived by the receiver of link m, γ_m, is given by:

$$\gamma_m = \frac{P_m \mathcal{A}(X_m, Y_m)}{\sum_{l \in \mathcal{L}, l \neq m} P_l \mathcal{A}(X_l, Y_m) + \mathcal{N}_m f} \qquad (1)$$

where Y_m denotes the location of the receiver of link m, $\mathcal{A}(x, y)$ denotes the channel attenuation from point x to point y, $\mathcal{N}_m$ denotes the power spectral density of the thermal noise at the receiver of link m, and f denotes the frequency bandwidth of the channel. The data-rate of link m, C_m, depends on the modulation and coding scheme used at the physical layer. A packet reception at the data-rate C_m is successful, provided that throughout the duration of the packet transmission:

$$\gamma_m > \theta_m \qquad (2)$$

where γ_m is the SINR defined by (1), and θ_m is an SINR threshold corresponding to an acceptable BER, depending on the modulation scheme. If at any point during the packet transmission (2) does not hold, then there is a collision. In order to simulate MAC protocols which employ physical carrier-sensing, we use the following model. A node located at Y will sense the channel busy if

$$\sum_{l \in \mathcal{L}} P_l \mathcal{A}(X_l, Y) + \mathcal{N}_m f > \theta_{cs} \qquad (3)$$

where θ_{cs} is the carrier-sensing threshold.

B. Capture Threshold Model

The capture threshold model can be viewed as a simplification of the above model as described in (1-3). It makes use of three thresholds which we denote following the *ns2* terminology, as `RxThresh`, `CpThresh` and `CsThresh`. The so-called receive threshold, `RxThresh` and the capture threshold `CpThresh` are both analogous to the SINR threshold θ (dropping the link subscript) described above. Using the earlier notation, a packet reception on a link m at the data-rate C_m is successful, provided that during the packet transmission:

$$P_m \mathcal{A}(X_m, Y_m) > \mathtt{RxThresh}_m \qquad (4)$$

$$\frac{P_m \mathcal{A}(X_m, Y_m)}{P_l \mathcal{A}(X_l, Y_m)} > \mathtt{CpThresh}_m \quad \forall l \in \mathcal{L}, \, l \neq m \qquad (5)$$

For carrier-sensing, a node at Y will sense the channel busy if

$$P_l \mathcal{A}(X_l, Y) > \mathtt{CsThresh} \text{ for some } l \in \mathcal{L} \qquad (6)$$

where `CsThresh` is the carrier-sensing threshold analogous to θ_{cs} as in (3).

[2]Note that the SINR to bit error rate (BER) mapping is based on the Gaussian approximation of multi-access interference, motivated by the Central Limit Theorem. In practice, this approximation may sometimes not hold (for instance, see [13]), in which case the BER may depend on quantities other than the SINR as well, depending on the modulation scheme. However, we feel that assuming BER as a function of the SINR is a reasonable practical approach.

C. Differences in Conflict Relationships

In order to understand the differences in the conflict relationships imposed by the two models among the wireless links, let us consider isotropic path loss, with $\mathcal{A}(x,y) = (\frac{|x-y|}{d_0})^{-\beta}$ where d_0 is a constant, and β is the path loss exponent. Let us first show how the capture threshold model is equivalent to the protocol model [4], under isotropic path loss, when all the nodes use the same transmit power. According to the protocol model, a packet transmission on link m is successful, provided that for each interfering link $l \in \mathcal{L}$, $l \neq m$, we have

$$|X_l - Y_m| > (1+\Delta)|X_m - Y_m| \qquad (7)$$

where Δ is a positive parameter. Equating the transmit powers in (5) and substituting for $\mathcal{A}(.,.)$, we get by comparing with (7) that the two models are equivalent with $\Delta = \texttt{CpThresh}^{1/\beta} - 1$.

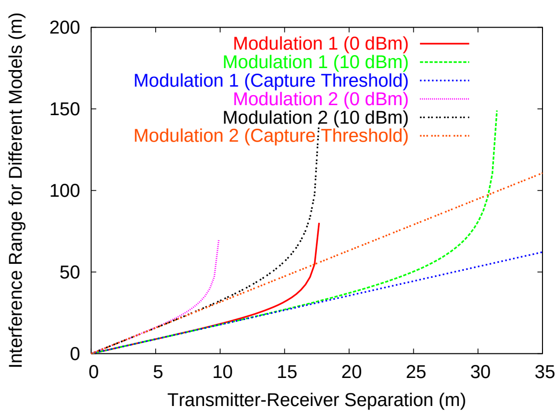

Fig. 1. Interference Range vs. Transmitter-Receiver separation

Now, let us define the *interference range* to be the minimum distance from the receiver of an active link, at which a *single* interfering node would not cause a collision by transmitting. Under the capture threshold model, as can be seen from (5) and (7), the interference range is just $\texttt{CpThresh}^{1/\beta}$ times the transmitter-receiver separation, independent of the transmit power level. Under the additive interference model, it can be calculated using (1-2), by considering interference from a *single* link as follows:

$$|X_l - Y_m| > \left(\frac{1}{\theta_m |X_m - Y_m|^{\beta}} - \frac{\mathcal{N}_m f}{P_m d_0^{\beta}}\right)^{-1/\beta} \qquad (8)$$

which is evidently not a linear function of the transmitter-receiver separation. Figure 1 shows an illustration for β=4. The channel attenuation and overall noise power are taken from Table I. We consider two transmit power levels, 0dBm and 10dBm, and two modulation schemes, modulation 1 with an SINR threshold of 10 dB, and modulation 2 with an SINR threshold of 20 dB. For the capture threshold model, we take CpThresh to be identical to the SINR threshold for a given modulation scheme.

TABLE I
PHYSICAL CHANNEL PARAMETERS

Parameters	**Values**
Overall Channel Bandwidth	22 MHz
Overall Transmit Power	0 dBm (1 mW)
Signal Attenuation (in dB) as a function of distance d	-40 dB - 40$\log(d)$
Overall Noise Power	-100 dBm
Carrier-sensing Threshold	6 dB over noise power
SINR Threshold for 1 Mbps	12 dB
SINR Threshold for 2 Mbps	15 dB
Busy-tone Channel Bandwidth	11 kHz
Control Channel Bandwidth in DUCHA	6.6 Mhz (30%)

As can be inferred from Figure 1, the capture threshold model is quite close to the additive interference model when the transmitter is close enough to the receiver, provided there is a *single interferer*. However, as the transmitter-receiver separation increases, approximating the interference range as a multiple of the transmitter-receiver separation, is no longer accurate. In particular, for very "weak" links (*i.e.*, when the receiver can just barely decode the signal from the transmitter), the interference range can be arbitrarily high.

Another difference between the two models, is that the conflict relationships between the wireless links are not *binary*. Specifically, the capture threshold model leads to *binary* conflict relationships among the wireless links (*i.e.*, any two links either interfere with each other, or they can be active simultaneously, regardless of the other on-going signal transmissions). On the other hand, under the additive interference model, even though a set of links may each not interfere with a given link, they could still collectively cause a collision. Now let us consider the following case-studies to see how these differences between the models, manifest themselves.

IV. Case-study 1: MAC Protocols for Random Access Wireless Networks

In this section, we use a discrete-event simulator we developed, to simulate four MAC protocols,

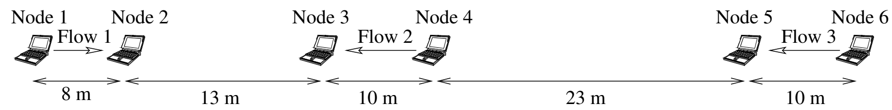

Fig. 2. A Simple Three Flow Topology

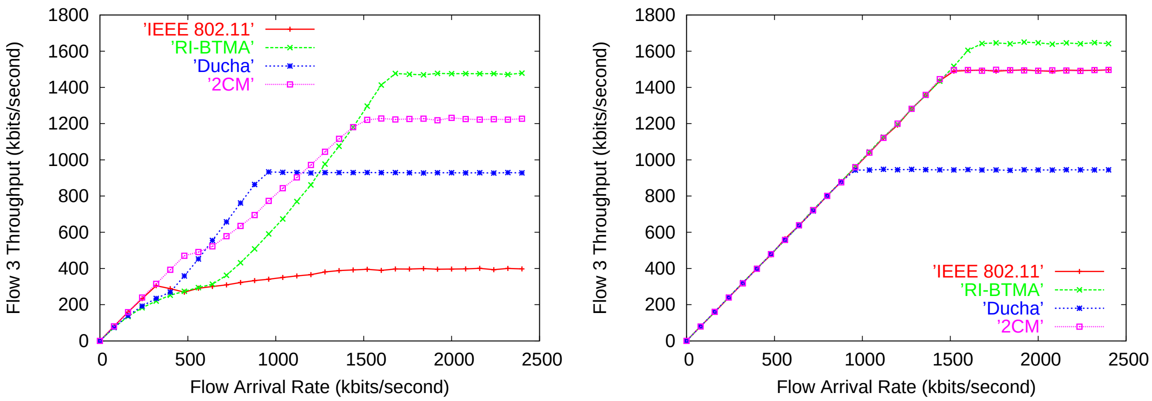

Fig. 3. Throughput of Flow 3 in Figure 2. Additive Interference model (left) and Capture Threshold Model (right).

viz., IEEE 802.11 [5], RI-BTMA (receiver-initiated busy-tone multiple access) [19], DUCHA (dual channel MAC) [20] and 2CM (two channel MAC) [6]. We simulate the four protocols under both the additive and the capture threshold models of interference, and compare the throughput observed. The 2CM protocol has been proposed in an earlier work of ours [6], from where we also borrow the example considered in this section (see Figure 2). However, the specific results presented here have *not* been presented elsewhere. For an introduction to the working of the four MAC protocols, the reader is directed to [6].

We start by providing the values used for the simulation parameters. These are summarized in Table I. A few explanatory notes follow. The protocol DUCHA uses a separate control channel for RTS-CTS, and a data channel for DATA packets. So the overall bandwidth has to be divided between the two channels. We indicate the proportion of bandwidth used for the control channel in Table I. The rest is used for the data channel[3]. The IEEE 802.11 standard [5] requires that the control packets RTS and CTS, be always transmitted at the base data-rate of 1 Mbps, while the DATA and ACK packets can be transmitted at higher data-rates. We adopt the same rule for simulating the other protocols as well. To be precise, we use only the base modulation and coding scheme to transmit the RTS and CTS packets. Thus, in the case of DUCHA, at the 2 Mbps data-rate, the control channel data-rate is 300 kbps (30% of the base data-rate, 1 Mbps), while the data channel data-rate is 1.4 Mbps (70% of the high data-rate, 2 Mbps).

We operate all the nodes with the same transmit power level (refer to Table I). For the capture threshold model, `CpThresh` is taken to be identical to the SINR threshold corresponding to the concerned data-rate, and `RxThresh` is taken to be `CpThresh` times the overall noise power, so that the additive and capture threshold model are consistent. The busy-tone channel bandwidth is taken to be 11 kHz. The transmit power and noise power levels are scaled down (from the values in Table I), in proportion to the bandwidth of the channel in which they are used. The carrier-sensing threshold is set to 6 dB over the noise power in the channel. We operate the network at the 2 Mbps data-rate. Note that we do not simulate any congestion control mechanism, instead allowing the data sources to generate packets at a given rate.

We consider the scenario in Figure 2 with distances, node labels and flow labels, as indicated. The following conflict relationships will help the reader understand the scenario better. Node 2 does not suffer from any collisions when receiving from node 1, due to their close proximity. Node 3 can receive an RTS packet from node 4, but not a DATA packet when node 1 is transmitting, since a DATA packet (at 2 Mbps) needs a higher SINR

[3]In the paper on DUCHA [20], the authors use about 22% of the bandwidth for the control channel for a network operating at 1 Mbps. We chose the values in Table I as they resulted in a better performance for DUCHA, in the example we considered.

for reception than an RTS packet (at 1 Mbps). But if both node 1 and node 6 transmit, node 3 cannot receive any packet from node 4. Node 5 can always receive an RTS packet from node 6. However, when node 4 transmits, node 5 cannot receive a DATA packet, again because a DATA packet requires a greater SINR. For busy-tone signals, node 4 cannot sense the busy-tone if node 2 or node 5 assert it individually, but it can sense the busy-tone when both node 2 and node 5 assert it simultaneously. Under the capture threshold model, the conflict relationships are much simpler to explain. Flow 2 and Flow 3 are immune to collisions, while Flow 1 cannot operate if Flow 2 is active.

The simulation experiment consists of simulating the network in Figure 2, with equal Poisson arrival rates for the three flows, for different arrival rates, and observing their long term throughput averaged over 1000 simulation seconds. We find that the throughput values already become stable around 100 simulation seconds. The throughput achieved by Flow 3, for all the protocols, under the two models, is shown in Figure 3. There are several points of note to be observed here. Firstly, note that not only does the capture threshold model overestimate the total throughput as compared to the additive interference model, but it also predicts *different qualitative* behavior. In particular, depending on the arrival rate, even the relative throughput rank of the protocols is different. Secondly, DATA packet collisions are *inevitable* under the additive interference model. On the other hand, they are *completely prevented* by RI-BTMA, DUCHA and 2CM, under the capture threshold model. Finally, the problem of DATA packet collisions, is further aggravated in RI-BTMA due to the following sequence of events, *only observable* under the additive interference model.

Refer to Figure 2, and consider flow 1 to be active (under RI-BTMA) which means that node 2 is asserting a busy-tone to prevent hidden nodes from transmitting. However, as mentioned earlier, it cannot be perceived at node 4. Now let both node 4 and node 6 transmit an RTS to node 3 and node 5, respectively, at about the same time. Since node 3 is exposed to the transmission from node 1, it does not receive the RTS and makes no reply. However, node 5 receives the RTS successfully and asserts a busy-tone. Due to the collective power of the busy-tones from node 2 and node 5, node 4 now begins perceiving a busy-tone (which it interprets as a CTS, in RI-BTMA) and begins transmitting the DATA packet. This results in a collision at both node 3 and node 5. Both these DATA packets are lost without even being retried, since RI-BTMA has no link layer acknowledgement mechanism. All of the above observations are further substantiated by the other examples considered in [6].

To conclude, not only is the capture threshold model optimistic in predicting the throughput performance as compared to the additive interference model, but it also predicts different qualitative behavior, and fails to model certain phenomena which are inevitable in reality.

V. Case-study 2: Optimal Routing in Scheduled Wireless Networks

In this section, we consider the problem of optimal routing in *scheduled* wireless networks. A wireless network is termed *scheduled* if all the node transmissions are globally coordinated as to be completely collision-free. We consider two examples: one with a single data source, and the other which is a sensor network where all the nodes except the sink, are data sources. The routing is optimal in the sense of maximizing the throughput. We start with a formal statement of the problem. Note that here we present a simplified version of the optimal routing problem which we formulate and explore, in greater detail and generality, in a companion paper [7].

A. Problem Statement

We state the problem in the context of a sensor network. It is straightforward to see how it applies to the case of a single data source. Consider a sensor network with N sensors numbered $1, 2, \ldots, N$, and a sink numbered 0. All the nodes operate at the same transmit power level P, and use a single modulation and coding scheme with data-rate C, and SINR threshold θ. The channel attenuation $\mathcal{A}(x, y)$ is assumed to be known. We form a directed link $l = (a, b)$ from node a to node b, if node b can successfully decode a packet transmission from node a in the absence of interference (as per equations (1-2) in Section III). The links are numbered $1, 2, \ldots, L$, where L is the total number of links. Note that the set of links does depend on the specific choice of the transmit power level and modulation scheme.

We assume a central entity which can finely control the wireless link transmissions. Let us denote by $\underline{x} = \{x_1, x_2, \cdots, x_L\}$, a link activation vector. To be precise, $x_l = 1$ if link l is active, and $x_l = 0$ otherwise. The set of simultaneously active links is then given by $\mathcal{L} = \{l | x_l = 1\}$. Under the configuration $\underline{x}$, not all links may be feasible. In particular, the data-rate of link l under $\underline{x}$, denoted by $\underline{c}(\underline{x}) = \{c_1(\underline{x}), \cdots, c_L(\underline{x})\}$, is given as follows. For the link l, $c_l(\underline{x}) = C$, provided $x_l = 1$ and the SINR $\gamma_l > \theta$ (as per equations (1-2) in Section III). Otherwise, $c_l(\underline{x}) = 0$. Let X denote the set of *conflict-free* configurations defined by

$$X = \{\underline{x} | c_l(\underline{x}) = C, \text{if } x_l = 1\} \quad (9)$$

In other words, under X every active link is successfully activated, getting a data-rate of C, *i.e.*, there are no collisions.

Now let $\underline{\alpha}$ denote a schedule of conflict-free configurations, and let $A(X)$ be the set of all schedules defined as below:

$$\underline{\alpha} = \{\alpha_{\underline{x}}; \underline{x} \in X\};\ A(X) = \{\underline{\alpha} \geq 0 | \sum_{\underline{x} \in X} \alpha_{\underline{x}} = 1\} \quad (10)$$

Under the schedule $\underline{\alpha}$, the configuration $\underline{x}$ is scheduled for the fraction of time $\alpha_{\underline{x}}$. Then, r_l, the amount of data that can flow on the directed link l, satisfies $r_l \leq \sum_{\underline{x} \in X, x_l = 1} C\alpha_{\underline{x}}$. We are interested in selecting a schedule $\underline{\alpha}$ that maximizes the rate λ at which the sensors can produce data about the environment. Formally stated, we want to:

$$\begin{gathered} \text{maximize } \lambda \quad (11) \\ \text{subject to} \\ \sum r_{(n,k)} - \sum r_{(j,n)} \geq \lambda \text{ for } 1 \leq n \leq N \\ r_l \leq \sum_{\underline{x} \in X, x_l = 1} C\alpha_{\underline{x}} \text{ for } 1 \leq l \leq L \\ \underline{\alpha} \in A(X),\ \underline{r} \geq 0,\ \lambda \geq 0 \end{gathered}$$

where $\underline{r} = \{r_1, r_2, \cdots, r_L\}$. The first constraint is just the flow conservation constraint which states that the amount of data leaving a sensor n is less than or equal to the amount of data arriving at the sensor. The second constraint states the amount of data flowing on the link l cannot exceed the capacity provisioned for it under the schedule $\underline{\alpha}$. Finally, the third constraint states that $\underline{\alpha}$ is a valid schedule, and that all the other variables are non-negative.

We solve the problem (11) numerically for the two scenarios that follow.

B. A Single Source Illustration

We consider the case of a single data source. The problem (11) is modified by setting the right-hand side of the flow conservation constraint to zero, for all the nodes except the source node. The topology and the data source and destination, are as shown in Figure 4. The distances between the nodes are $8m$ along the vertical and horizontal branches, and $8\sqrt{2}m$ along the diagonal. All nodes use the same power level, -7.85dBm, and the same modulation scheme with SINR threshold 10 dB. The channel attenuation and overall noise power, are as in Table I. For this power and modulation scheme, the vertical, horizontal, and diagonal links are feasible.

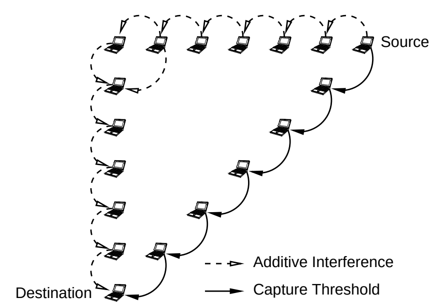

Fig. 4. Optimal Routing for Additive and Capture Threshold Interference Models

The optimal routing under the two models for this scenario is as indicated in Figure 4. As can be seen, the capture threshold model favors the shortest path, while the additive interference model favors longer paths. Note that the additive interference model also uses multi-path routing (top left corner) where the only diagonal link is used. The optimal (normalized) throughput under the additive model is 0.29, while under the capture threshold model it is 0.33. But, if the optimal routing of the capture threshold model, is used under additive interference, it only yields a throughput of 0.17.

In order to understand why the two models pick different paths to maximize throughput, refer to Figure 1. For the diagonal links, the interference range as predicted by the additive interference model is very high (steep part of the curve in Figure 1), whereas it is considerably lower under the capture threshold model (interference range is linear for the capture threshold model, as in Figure 1). So the capture threshold model optimistically selects the diagonal links, while the additive model rightly avoids them.

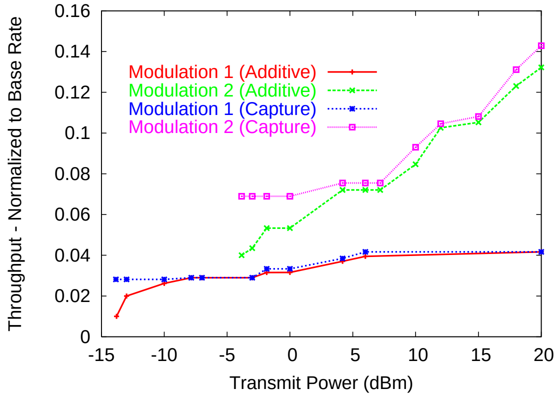

Fig. 5. Optimal Throughput vs. Transmit Power

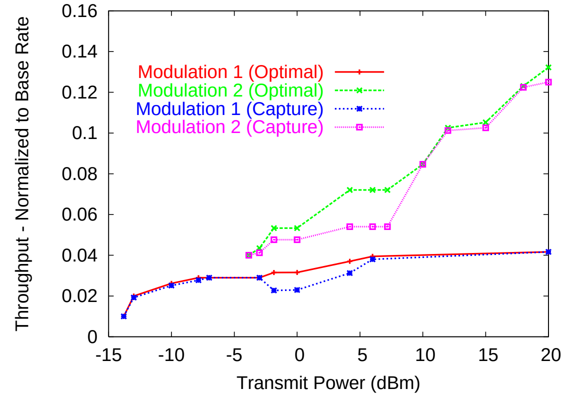

Fig. 6. Throughput of Capture Threshold Optimal Routing

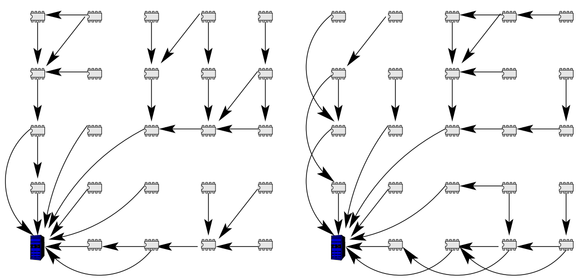

Fig. 7. Optimal Routing for Modulation 1 at 4.185 dBm. Additive Interference model (left) and Capture Threshold model (right).

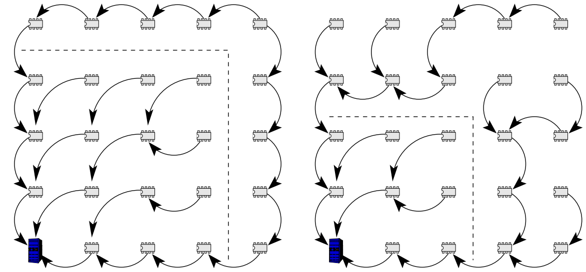

Fig. 8. Optimal Routing for Modulation 2 at 4.185 dBm. Additive Interference model (left) and Capture Threshold model (right).

C. A Grid Sensor Network

Next we consider a sensor network deployed in the form of a 5x5 grid (see Figure 7). The sink is at the bottom left corner. Again, we assume that all nodes use the same transmit power, and the same modulation scheme. The separation between adjacent nodes is $8m$. The channel attenuation and overall noise power, are as in Table I. Figure 5 shows the optimal throughput as a function of the transmit power, for different modulation schemes, for both the interference models. Modulation 1 and 2 are schemes which have data-rates of 1 and 4 (normalized), and SINR thresholds of 10dB and 20dB. As can be expected, the capture threshold model overestimates the optimal throughput, but is actually quite close in predicting the trend. Again, referring to Figure 1, we know that for weaker links, the capture threshold model is less accurate, and therefore the greater the discrepancy at the left ends of the two sets of curves.

However, since the capture threshold model is still able to follow the trend of optimal throughput quite closely, a natural question that arises is: how good is the optimal routing predicted by the capture threshold model when implemented under the additive model? This is shown in Figure 6. The curves marked "capture" are obtained by imposing the capture threshold routing, on the network, under the additive interference model. The way we obtain these results, is by solving problem (11) with those r_l variables artificially set to zero, which correspond to links that are not used under the capture threshold routing. As can be seen, the capture threshold model can predict some suboptimal routes, resulting in poorer throughput, though not terribly so.

Let us look at two examples to understand how the optimal routing differs. Figure 7 shows that the capture threshold routing uses longer links and shortest-path routes more optimistically. However, such a simple insight is not strictly true. In Figure 8, the capture threshold model uses more non-shortest path routes than the additive interference model (beyond the region indicated by the dotted line). Note that both models make use of multi-path routing. To conclude, the capture threshold model is not only optimistic, but it can also be misleading, and there is no consistent relationship between its predictions and that of the additive model.

VI. Conclusions

A wireless network fundamentally differs from a wireline network in the intrinsic difficulty in abstracting its physical layer in terms of few simple per link parameters. In contrast with wired links which have fixed capacities independent of one another, in a wireless network, the links suffer from mutual interference creating a need to model the wireless channel interference. Several works in the literature have made use of simplified interference models. We have looked in detail at one such model, the capture threshold model, which determines packet reception by comparing the desired signal strength with interference from a single node at a time, rather than cumulatively. We find that both in the case of random access networks, and scheduled networks, the capture threshold model can produce significantly different results as compared to an additive interference based model. Therefore, the importance of using a realistic model cannot be overemphasized.

Our work can be viewed as complementary to the work in [17], [10] which investigate the limitations of using isotropic path loss as a channel attenuation model. A complete model would include both realistic attenuation models, as well as accurate models for SINR calculation and determining packet reception. Such a complete model would without doubt be the *right* model, from a simulation standpoint. From the analytical perspective, such a full-fledged channel model would be useful in determining the minimum level of detail that needs to be modeled to obtain consistent and accurate results. Our work is a useful step in this direction since it shows that an additive interference model is the minimum level of detail that should be modeled.

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