Joint Resource Allocation and User Association for Heterogeneous Wireless Cellular Networks

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***Abstract*—We propose a unified static framework to study the interplay of user association and resource allocation in heterogeneous cellular networks. This framework allows us to compare the performance of three channel allocation strategies: *Orthogonal deployment*, *Co-channel deployment*, and *Partially Shared deployment*. We have formulated joint optimization problems that are non-convex integer programs, are NP-hard, and hence it is difficult to efficiently obtain exact solutions. We have, therefore, developed techniques to obtain upper bounds on the system's performance. We show that these upper bounds are tight by comparing them to feasible solutions. We have used these upper bounds as benchmarks to quantify how well different user association rules and resource allocation schemes perform. Our numerical results indicate that significant gains in throughput are achievable for heterogeneous networks if the right combination of user association and resource allocation is used. Noting the significant impact of the association rule on the performance, we propose a simple association rule that performs much better than all existing user association rules.**

***Index Terms*—Heterogeneous cellular networks, user association, channel allocation, interference management.**

I. INTRODUCTION

Heterogeneous cellular networks (Hetnets) are composed of macro base stations (BS) overlaid with a set of low-power BSs of different types, including pico (also called small cells in the literature), femto, and relay BSs. Hetnets are designed to improve spectral efficiency per unit area [1]. The mixture of different BSs with different power levels and different cell sizes can lead to significant gains in performance by offering higher spatial reuse, by eliminating coverage holes, and by creating hot-spots. The LTE-Advanced standard, for example, proposes improvement to network-wide spectral efficiency by employing a mix of low-power BSs [2], [3].

Typically, an operator will place low-power BSs at strategic points to improve performance while keeping the infrastructure cost low. Hence, a user might not always be in the coverage area of a low-power BS. This being said, users should try to associate with low-power BSs if they can, to improve spectral efficiency. This association should improve the throughput and result in a higher spatial reuse, if resource allocation and interference management mitigate interference among low-power BSs and there are enough resources at the low-power BSs to serve all the users in their vicinity. Therefore, intelligent user association, resource allocation, and interference management schemes are needed to achieve gains in performance, and the interplay between these schemes have to be studied carefully. We now discuss these schemes in more details.

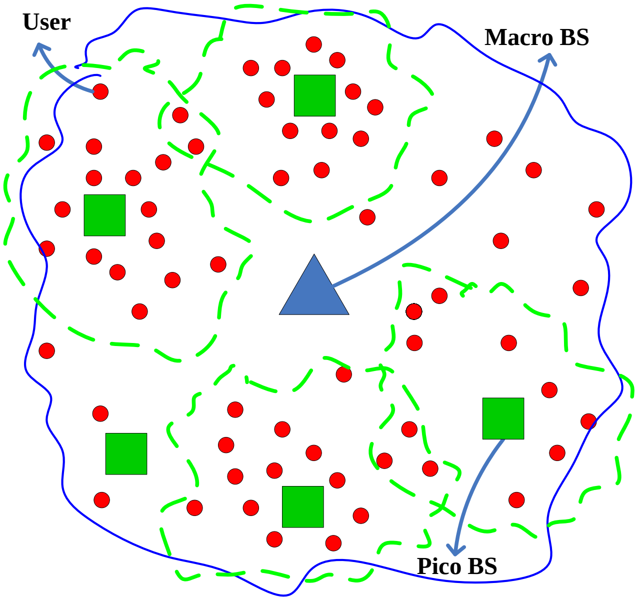

Fig. 1. A Hetnet comprising one macro BS (the triangle), and many pico BSs (the squares), and some users non-uniformly distributed in the area. The coverage area of the macro and the pico BSs are shown in plain and dotted lines, respectively.

User Association: This defines a set of rules for assigning users to the different BSs available in the system. A decision to associate a user with one BS will affect the throughput seen by that user, as well as the throughput of the other users associated with that BS. In conventional homogeneous cellular networks, and also in LTE Release-8 [2], user association is based on downlink received signal strength. Many association rules have been proposed that perform better than the conventional rule in Hetnets (e.g., [6], [7]); however, it is not clear which one is the best option since each study is based on a different resource allocation scheme and a different set of assumptions.

Resource Allocation and Interference Management (RAIM): Typically, Hetnets are based on OFDM[1], and hence one of the resources to distribute among the different BSs is sub-channels. Another important resource is transmit power. Given a fixed number of channels and a fixed total transmit power (possibly different) at each BS, a RAIM scheme deter-

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[1] We assume that the Hetnet as a whole is allocated a frequency band that is divided into M orthogonal sub-channels where each sub-channel has a bandwidth b. We will use the term channel and sub-channel interchangeably in the paper.

mines how to allocate the channels among the BSs, and how to use the power budget on the allocated channels at each BS. Hence, in its most complex form, a RAIM scheme can be seen as a centralized scheduling deciding which BSs should transmit to whom, on which channels, and with what transmit power, at each time. Even in a static scenario where channel gains are known and fixed, and the association is given, this problem is not tractable due to the very large number of variables. In its simplest form, a RAIM scheme might allow each BS to transmit at all time on all sub-channels (and to cope with the resulting interference) using the same power on each channel. In that case, for a given association rule, each BS can schedule locally its own users without the need for any coordination with the other BSs. Clearly, even in this simple case, one expects different performance for different association rules. In our study, *we focus on RAIM schemes that do not require fine coordination among BSs*, i.e., the schemes determine the number of channels that each BS can use and each BS then uses these channels at all time with the maximum allowed transmit power (distributed over these channels) to schedule its users.

There is a need to develop a *unified framework* to analyze, compare, and evaluate the performance of different resource allocation schemes when user association is either computed optimally or performed via the use of simple rules. Our framework is centralized and static since we consider a snapshot of the system both in terms of user deployment and channel gains. This framework allows us to perform an offline study of different combinations of RAIM and user association schemes to select the best performing ones.

We consider three RAIM schemes. In the first one, all BSs use all the available M sub-channels (we refer to this scheme as *Co-channel deployment* (*CCD*)). In the second one, all pico BSs share K of these sub-channels while the macro BS uses the other $M - K$ (we refer to this scheme as *Orthogonal deployment* (*OD*)). In the third one called *Partially Shared deployment* (*PSD*), all pico and macro BSs share K of these sub-channels while the macro BS uses the other $M - K$ sub-channels. To reduce interference among the macro and pico BSs over the K sub-channels, the macro BS uses a lower power on the K sub-channels.

We study a scenario comprising one macro BS and several pico BSs by focusing on the downlink and on RAIM schemes that do not require fine coordination among BSs. Our contributions are:

1) We formulate a centralized static unified framework to analyze and compare several combinations of association rules and RAIM schemes. We consider three RAIM schemes: CCD, OD, and PSD. For *CCD*, we formulate an optimal user association problem. For *OD* and *PSD*, we formulate an optimal joint user association and resource allocation problem. For these three problems we consider an objective function corresponding to proportional fairness (PF) among all users in the system (we call this global PF). These three problems are multi-purpose in that, they can be used to compute the optimal association for each RAIM scheme under consideration (along with the optimal channel allocation for *OD* and *PSD*), which gives us a benchmark (i.e., an upper bound) on the performance to be expected for each scheme. In the case of *CCD*, the problem can be used to compute the performance of a given association rule. In the case of *OD* and *PSD*, the problem can be used to compute for a given association rule the optimal splitting and the corresponding performance.
2) We show how the global proportional fairness criteria yields to a solution in which each BS schedules its users using local proportional fairness.
3) Although the problems in their more general form are non-convex integer programs, we are able to develop numerical techniques to compute tight upper bounds on the performance for small to large systems.
4) We use the numerical results to compare the three RAIM schemes when the association is optimal. We find that under our assumptions (especially the one on the absence of coordination among BSs), *OD* and *PSD* perform significantly better than *CCD*.
5) We then focus on *PSD* and study the impact of different parameters and how different simple association rules perform. In particular, we propose a very simple association rule and show that it works better than the existing association rules.

This study is a first step to systematically compare different user association rules and resource allocation schemes for Hetnets. It shows the critical impact of the association rule and the resource allocation scheme in achieving good performance.

The paper is organized as follows: Section II presents the related work. The system model is introduced in Section III. In Section IV, the formulations of the optimization problems are presented as well as the relationship between local proportional fairness and global proportional fairness and our solution techniques. In Section V, we introduce three simple user association rules that we study and compare to the computed upper bound. Numerical results are provided in Section VI. All the proofs are presented in the Appendix.

II. RELATED WORK

OD, *PSD*, and *CCD*, have been proposed in 3GPP to share resources between macro and pico tiers [5]. *OD* mitigates interference among different classes of BSs since they are allocated orthogonal sub-channels. In *PSD*, capacity gains can be achieved by using low-power BSs without affecting the coverage of the macro BSs. Note that to reduce interference among macro and low-power BSs on the shared spectrum, macro BSs transmit with reduced power on the shared frequency resources. This solution provides an efficient way of using resources without affecting coverage area of macro BSs. In *CCD*, all BSs have access to the whole set of channels. This solution is considered more efficient for systems with limited spectrum since it avoids spectrum partitioning, and for systems in which *PSD* is not supported by user equipments. *CCD* results in high interference among macro and low-power BSs so that the coverage of low-power BSs is reduced and their capacity gains are diminished if no interference management technique is deployed in the system. In [8], the authors explore the performance of *OD* and *CCD* with the assumption that the system uses the conventional association rule in which a user associates with the BS that provides the highest downlink signal power. The authors show, via simulation, that *CCD* achieves a higher average throughput when the channels are equally divided between the macro and pico BSs.

In [4], a new association rule called "Range Expansion" is proposed. It adds a bias to the reference signal received power received from pico BSs to artificially extend their coverage [5]. The bias can be selected such that users associate with the BS with the minimum path loss. This gives a simple association rule called "range extension" (RE) [6]. In [6], the authors consider the effect of this user association on the network's throughput for a given fixed partitioning of resources between the macro and some pico BSs. They show by simulation that RE can improve the throughput when compared to the conventional association rule. We will compare RE to other association rules in this paper.

The work closest to ours is [9] in which user association and resource allocation among macro and pico BSs are studied. An OFDM system is considered in which the total bandwidth is divided into M sub-channels. Power and scheduling time (on a per sub-channel basis) are the resources that are allocated among different BSs. A global high level scheduling problem is formulated to maximize at each time-slot the sum of the logarithm of the rates as a function of several variables, including power levels, scheduling, sub-channels and user association. However, since this problem is a very large combinatorial problem, they propose heuristic algorithms for adaptive user association and resource partitioning. The performance of these heuristic algorithms are compared to the performance of systems using different simple user association rules (i.e., the conventional one and RE) and resource partitioning schemes including *CCD* and *OD* (in which the sub-channels are equally divided between the macro and pico BSs) for resource partitioning.

In our preliminary work [10], we develop a *unified framework* to analyze, compare, and evaluate the performance of different user association rules in Hetnets using max-min scheduling for *OD* only. In this paper, we assume that the system uses proportional fairness scheduling. We formulate the problem of joint user association and resource allocation for *OD*, *PSD*, and *CCD*. The proposed problems are Integer non-convex programs that are NP-hard. Unlike the proposed problem in [10], we cannot obtain exact solutions to such problems.

Extensive work has been done on user association and resource allocation schemes in Hetnets, but none of these works can be used as a unified benchmark to compare the performance of existing user association rules and resource allocation schemes.

III. SYSTEM MODEL

We consider a communication system composed of one macrocell (cell 0) overlaid with X pico BSs (cells $j = 1, \cdots, X$) that are identical in terms of transmit power, antenna gain, and backhaul capacity (see Fig. 1). We study the downlink and make the following assumptions:

- The system is an OFDM system with M "data" sub-channels, each of bandwidth b.
- Each user can associate with only one BS.
- Each pico BS is connected to the macro BS via a high capacity wired backhaul.
- All BSs are *active* at all time, i.e., there is no time at which a BS is not transmitting and a BS uses all its transmit power at all time.
- There are N fixed users in the system. All users' information, including the channel gains (assumed to be flat, i.e., the same on each sub-channel for a given (user,BS) pair), are available so that the SINR (Signal to Interference and Noise Ratio) to each user from each BS can be computed.
- The rate function $f(\cdot)$ for each BS is known so that given the SINRs, users' rates from all BSs can be computed. We do not make any restricting assumptions on f. We assume f is the same for each BS (though our framework does not depend at all on this assumption).

Power and sub-channels are the resources that we allocate to the different BSs, so that our global objective function is maximized. We study different resource allocation and interference management (RAIM) schemes, including three channel allocation strategies and one type of power allocation scheme well studied in the literature.

A. Channel Allocation

We study three channel allocation schemes as follows:

- *Co-channel deployment (*CCD*):* Each BS transmits on all the sub-channels.
- *Orthogonal deployment (*OD*):* K sub-channels are dedicated exclusively to the pool of pico BSs and $(M - K)$ sub-channels are dedicated to the macro BS. The K sub-channels are divided among the pico BSs based on conventional frequency reuse [11], i.e., given reuse factor u the K sub-channels are equally divided among the pico BSs such that each pico BS is granted a group of $\frac{K}{u}$ sub-channels and co-channel BSs (i.e., BSs using the same group of sub-channels) are spaced a couple of cells away. Given a reuse factor u, there are multiple ways (or patterns) to assign sub-channels to pico BSs. Two examples are provided in Section VI. In the following, we will restrict our study to a limited set of reuse factors (typically $u \in \mathcal{U} = \{1, 2, 3\}$) and a limited set of preselected reuse patterns, i.e., the set $\mathcal{P}(u)$ of patterns is given and small. By choosing u and a reuse pattern $p(u) \in \mathcal{P}(u)$ carefully, we can mitigate co-channel interference between the pico BSs at the expense of reducing the bandwidth at each pico BS.
- *Partially Shared deployment (*PSD*):* K sub-channels are shared by the macro and pico BSs and the other $(M-K)$ sub-channels are dedicated to the macro BS. The K sub-channels are equally divided among the pico BSs based on a given reuse factor $u \in \mathcal{U}$ and reuse pattern $p(u) \in \mathcal{P}(u)$ while the macro BS transmits over all the K sub-channels.

B. Power Allocation

We assume that the total transmit power of the macro (P_m) and pico (P_p) BSs are fixed and known. For *CCD* and *OD*, we assume that the power budget of a BS is shared equally among all channels allocated to this BS. For *PSD*, we assume that the macro BS uses the same transmit power budget P_p on the K channels shared with the pico BSs, and that it uses $(P_m - P_p)$ on the other $(M - K)$ sub-channels [12].

C. Physical Link Model

Let $\mathcal{N}$ and $\mathcal{B}$ denote the sets of users and BSs in the system, respectively. The SINR of user $i \in \mathcal{N}$ from BS $j \in \mathcal{B}$ on each sub-channel (i.e., on the downlink) can be written as:

$$\gamma_{ij}^{(c)} = \frac{P_j^{(c)}\ G_{ij}}{N_0 + \sum_{h \in \mathcal{I}_j} P_h^{(c)}\ G_{ih}} \tag{1}$$

where $\mathcal{I}_j$ is the set of BSs transmitting on the same set of sub-channels (not including j), $P_j^{(c)}$ is the transmit power of BS j on each of its sub-channels, N_0 is the additive white Gaussian noise power, and G_{ij} is the flat gain between user i and BS j that accounts for the path loss, shadow fading, antenna gain, and equipment losses. Note that given a RAIM scheme (i.e., CCD, OD, or PSD), $\mathcal{I}_j$ the set of BSs that use the same set of sub-channels as well as the transmit power of each BS on each sub-channel, $P_j^{(c)}$, can be determined. Then $\gamma_{ij}^{(c)}$ can be calculated for all $i \in \mathcal{N}$ and $j \in \mathcal{B}$.

As mentioned earlier, we assume there is a mapping function $f(\cdot)$ that maps the SINR to the corresponding rate in bps (bit/second), i.e., $r_{ij}^{(c)} = f(\gamma_{ij}^{(c)})$. Next, we formulate our optimization problems, one for each of the three channel allocation schemes presented above.

IV. PROBLEM FORMULATIONS AND SOLUTION TECHNIQUES

As briefly mentioned earlier, we select proportional fairness as our global objective function, i.e., we maximize $\sum_i \log(\lambda_i)$ where λ_i is the throughput of user i. To compute λ_i, let r_{ij} denote user i's link rate from BS j (i.e., $r_{ij} = |\mathcal{C}_j| \times f(\gamma_{ij}^{(c)})$ where $\mathcal{C}_j$ is the set of (flat) sub-channels allocated to BS j) and let α_{ij} be the proportion of time that user i is scheduled on the downlink by BS j. We assume that a BS allocates all its sub-channels to a user at the same time (which is a reasonable assumption if the channels are flat). Let $x_{ij} = 1$ if user i is associated with BS j, and let it be 0, otherwise. Hence, for all $i \in \mathcal{N}$, $\sum_{j \in \mathcal{B}} x_{ij} = 1$. Note that we implicitly assume that each user i can hear at least one BS with a non-zero rate, i.e., there are no non-covered users in the system. Hence, $\lambda_i = \sum_{j \in \mathcal{B}} (x_{ij}\alpha_{ij})\, r_{ij}$. Note that each BS j allocates all its time among its associated users and hence, $\sum_{i \in \mathcal{N}} (x_{ij}\alpha_{ij}) = 1$.

We begin with the formulation for *Co-channel deployment*. In this case, the problem is only one of optimal association and scheduling, i.e., the variables are the $\{x_{ij}\}$'s and the $\{\alpha_{ij}\}$'s. The problem can be formulated as follows: given the *CCD* channel allocation, the M channels, the channel gains for the N fixed users, the rate function $f(\cdot)$, the transmit powers, compute $\{x_{ij}\}$ and $\{\alpha_{ij}\}$ so as to maximize the global proportional fairness objective:

$$\mathbf{P_{CCD}}: \quad \max_{\{x_{ij}\},\{\alpha_{ij}\}} \quad \sum_{i \in \mathcal{N}} \log(\lambda_i)$$

$$\text{s.t} \quad \lambda_i = \sum_{j \in \mathcal{B}} (x_{ij}\alpha_{ij})\, r_{ij}\ , \quad \forall i \in \mathcal{N} \tag{2a}$$

$$\sum_{j \in \mathcal{B}} x_{ij} = 1\ , \quad \forall i \in \mathcal{N} \tag{2b}$$

$$\sum_{i \in \mathcal{N}} (x_{ij}\alpha_{ij}) = 1\ , \quad \forall j \in \mathcal{B} \tag{2c}$$

$$r_{ij} = M \times f(\gamma_{ij}^{(c)})\ , \quad \forall i \in \mathcal{N},\ \forall j \in \mathcal{B} \tag{2d}$$

$$0 \le \alpha_{ij} \le 1,\ x_{ij} \in \{0,1\}\ , \quad \forall i \in \mathcal{N},\ \forall j \in \mathcal{B} \tag{2e}$$

where $P_j^{(c)} = \frac{P_m}{M}$ if $j = 0$ and $P_j^{(c)} = \frac{P_p}{M}$ otherwise.

We assume that the backhaul network is not the bottleneck. More precisely, let C_j denote the capacity of the wired backhaul between pico BS j and the macro BS. For each feasible solution $\{x_{ij}\}$, we need to have $\sum_{i \in \mathcal{N}} x_{ij}\lambda_i \le C_j$ for all $j \in \mathcal{B}'$ where $\mathcal{B}'$ denotes the set of pico BSs. If C_j is sufficiently large, i.e., $\sum_{i \in \mathcal{N}} x_{ij}\lambda_i \ll C_j$ for all feasible solutions $\{x_{ij}\}$ to $\mathbf{P_{CCD}}$, these constraints will be satisfied, and they can be removed from the problem.

Before discussing this problem in more details, we formulate the problem of optimal user association and resource allocation for *Orthogonal deployment* that allocates the first K sub-channels to the pico BSs and the rest to the macro BS. Given $(u, p(u))$, each pico BS will be assigned $\frac{K}{u}$ sub-channels, and the set of pico BSs using the same set of sub-channels will be determined by the reuse pattern $p(u)$. Note that $P_j^{(c)} = \frac{uP_p}{K}$ for $j \in \mathcal{B}'$, and $P_0^{(c)} = \frac{P_m}{M-K}$.

In the case of *OD*, we optimize the same objective function as for *CCD* with respect to the following variables: K, $\{x_{ij}\}$, $\{\alpha_{ij}\}$, and $\{u, p(u)\}$. Note that the effect of $p(u)$ is implicit in $\gamma_{ij}^{(c)}$. The problem can be formulated as follows: given the *OD* channel allocation, the M channels, the channel gains for the N fixed users, the rate function $f(\cdot)$, the transmit powers, a set of reuse factors $\mathcal{U}$, and a set of reuse patterns $\mathcal{P}(u)$, compute K, u, $p(u)$, $\{\alpha_{ij}\}$, and $\{x_{ij}\}$ so as to maximize the proportional fairness objective:

$$\mathbf{P_{OD}}: \quad \max_{\substack{K,\{x_{ij}\},\{\alpha_{ij}\} \\ \{u,p(u)\}}} \quad \sum_{i \in \mathcal{N}} \log(\lambda_i)$$

$$\text{subject to} \qquad (2a),(2b),(2c),\ \text{and}\ (2e)$$

$$r_{i0} = (M-K) \times f(\gamma_{i0}^{(c)}),\ \forall i \in \mathcal{N} \tag{3a}$$

$$r_{ij} = \frac{K}{u} \times f(\gamma_{ij}^{(c)}),\ \forall j \in \mathcal{B}',\ \forall i \in \mathcal{N} \tag{3b}$$

$$u \in \mathcal{U},\ p(u) \in \mathcal{P}(u),\ K \in \{0, 1, \cdots, M\} \tag{3c}$$

For *PSD*, the macro BS transmitting on the K sub-channels can be considered as a new BS in the system. By doing this, we optimize the same objective function as for OD with respect to K, $\{x_{ij}\}$, $\{\alpha_{ij}\}$, and $\{u, p(u)\}$, and we obtain a problem $\mathbf{P_{PSD}}$ similar to $\mathbf{P_{OD}}$. Due to space limitations, we do not present the problem formulation for *PSD*.

Our objective is to solve these three problems exactly which is not going to be possible as we explain now. First, note that the proposed problem $\mathbf{P_{OD}}$ is a very complex problem. Some variables such as K, u and $p(u)$ are discrete while some others such as $\{\alpha_{ij}\}$ are continuous. Hence, it is hard to solve this problem as it is. Since $\mathcal{U}$ and $\mathcal{P}(u)$ are finite sets, and $K \in \{0, 1, \cdots, M\}$, a solution for $\mathbf{P_{OD}}$ can be obtained by solving $\mathbf{P_{OD}}$ iteratively for all possible values of K, u, and $p(u)$ and then selecting the best solution. In particular, let's define the optimal value of the objective function for $\mathbf{P_{OD}}$ for a given K, u, and $p(u)$ as $PF^{\star}(K, u, p(u))$. Hence, the solution for $\mathbf{P_{OD}}$ can be obtained by solving $\max_{\{K,u,p(u)\}} \{PF^{\star}(K, u, p(u))\}$. Let $\mathbf{P'_{OD}}$ and $\mathbf{P'_{PSD}}$ be the problems obtained by fixing K and $\{u, p(u)\}$. $\mathbf{P'_{OD}}$ (as well as $\mathbf{P'_{PSD}}$) reduces to a joint problem of optimal user association and scheduling (as is $\mathbf{P_{CCD}}$). These three problems are non-convex integer programs (IP) and are NP-hard [13]. Hence, it is impossible to obtain exact solutions to these problems. Our goal is to transform these problems into convex problems for which the relaxed programs can be solved efficiently (i.e., for which upper bounds can be computed). To do so we will need two steps. We will explain these steps for *OD*, but similar steps can be used for *PSD* and *CCD*. In the first step, we are going to show that $\mathbf{P'_{OD}}$ can be reduced to a pure optimal association problem by proving that for the optimal solutions, $x_{ij}\alpha_{ij} = \frac{x_{ij}}{N_j}$ where $N_j = \sum_{i\in\mathcal{N}} x_{ij}$ is the number of users associated with BS j. This means that the global PF criteria yields a solution based on local PF at each BS (i.e., each BS offers the same amount of time to all its users). In the second step, we will transform this pure optimal association problem into a non-linear convex program whose solutions provide tight upper bounds on the solutions of $\mathbf{P'_{OD}}$.

STEP 1 : As mentioned earlier, we focus on *Orthogonal deployment* although similar results hold for *CCD* and *PSD*. Assume each BS uses local PF scheduling. According to Lemma 1 [14], a BS assigns the same amount of time to its users.

Lemma 1. *[14] Let's assume there is one BS and all users have the same priority. Given resource allocation parameters including the number of sub-channels and the transmit power on each sub-channel, PF scheduling assigns equal proportion of time to all users.*

We can then formulate a new pure association problem called $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ as follows where we assume that each BS schedules using local PF:

$$\mathbf{P'}^{\ell}_{\mathbf{OD}} : \max_{\{x_{ij}\},\{N_j\}} \sum_{i\in\mathcal{N}} \log(\lambda_i)$$

$$\text{subject to} \qquad (2b), (2c)$$

$$\lambda_i = \sum_{j\in\mathcal{B}} \left(\frac{x_{ij}}{N_j}\right) r_{ij} \ , \quad \forall i \in \mathcal{N} \tag{4a}$$

$$x_{ij} \in \{0, 1\}, \ N_j = \sum_{i\in\mathcal{N}} x_{ij} \ , \quad \forall j \in \mathcal{B}, \quad \forall i \in \mathcal{N}, \tag{4b}$$

where all r_{ij}'s can be computed beforehand and used as inputs to the optimization problem.

We say that two problems are equivalent if and only if an exact solution of one is an exact solution of the other.

Theorem 1. *Given $\mathcal{B}$, $\mathcal{N}$, M, the channel gains, the rate function, the parameters of the* OD, *i.e., K, $u, p(u)$, $\mathbf{P'_{OD}}$ and $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ are equivalent.*

Based on this theorem, we work now with $\mathbf{P'}^{\ell}_{\mathbf{OD}}$. Note that $\mathbf{P}^{\ell}_{\mathbf{CCD}}$ and $\mathbf{P}^{\ell}_{\mathbf{PSD}}$ can be reduced to the *same* non-convex IP problem since their differences are all summarized in the r_{ij}'s that can be computed beforehand.

STEP 2 : To obtain an upper bound for $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ we could try to simply relax the integrality constraints on $\{x_{ij}\}$ (i.e., we assume that $0 \le x_{ij} \le 1$ for all i, j) and try to solve the relaxed problem. However, even after relaxing the integrality constraints in $\mathbf{P'}^{\ell}_{\mathbf{OD}}$, the problem remains non-convex. Note that non-convex programs cannot be solved exactly easily. Fortunately, the structure of $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ is such that we can reformulate it into an integer convex problem as follows. Noting that all x_{ij}'s are binary variables and $\sum_{j\in\mathcal{B}} x_{ij} = 1$ for all users, there exists only one value of j, i.e. $\bar{j}$, for which $x_{i\bar{j}} = 1$ (i.e., $x_{ij} = 0$, $\forall j \ne \bar{j}$). Therefore, the objective function in $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ can be rewritten as follows:

$$\sum_{i\in\mathcal{N}} \log\left(\sum_{j\in\mathcal{B}} \frac{x_{ij}}{N_j} \ \ r_{ij}\right) = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{B}} x_{ij} \ \log\left(\frac{r_{ij}}{N_j}\right) \ . \tag{5}$$

Using this property, $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ can be reformulated into a convex integer program and the relaxed program (with respect to the integrality constraints on $\{x_{ij}\}$) can be solved efficiently even for large systems since it is a convex problem. Note that this problem is convex, and hence it can be solved to the desired precision in polynomial time [15]. This enables us to obtain upper bounds on the performance of $\mathbf{P}^{\ell}_{\mathbf{CCD}}$, $\mathbf{P'}^{\ell}_{\mathbf{OD}}$, and $\mathbf{P'}^{\ell}_{\mathbf{PSD}}$ in terms of the global objective function, i.e., $\sum_i \log(\lambda_i)$, for large Hetnets that are composed of a large number of users, one macro BS, and many pico BSs. Although we are unable to show the tightness of these bounds analytically, we show numerically that $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ does provide tight upper bounds. We can verify the tightness of these upper bounds by finding a feasible solution for a given resource allocation and then comparing the corresponding performance metric $\sum_i \log(\lambda_i)$ for this feasible solution with the computed upper bound. To generate feasible solutions for a given RAIM, we will use simple association rules. It is important to note that the problems $\mathbf{P_{CCD}}$, $\mathbf{P_{OD}}$, and $\mathbf{P_{PSD}}$ can be used to provide the performance metric for a given association rule. Indeed, if the association rule is given, then the $\{x_{ij}\}$'s are given and the problems can then be solved easily. We will use this fact to compare the performance of several simple user association rules under our three resource allocation schemes, as shown in the sequel.

The purpose of our study is threefold: First, we want to compare the three resource allocation schemes, i.e., *CCD*, *PSD*, and *OD* not only in terms of the objective function, but also in terms of aggregate throughput, and minimum throughput in the system, i.e., the performance metrics are $\sum_i \log(\lambda_i)$, $\sum_i \lambda_i$, and $\min_i \{\lambda_i\}$. Operators are typically trying to trade-off fairness (usually using proportional fairness criteria), the total aggregate throughput which is a measure of the "capacity" of their system, and some criteria to take edge users' performance into consideration. We chose to use the minimum rate in the system as such a measure. Second, we want to study how different simple association rules perform as compared to the optimal solutions for these three resource allocation schemes. Finally, we want to study in more details the impact of some of the parameters of *PSD* which, under our assumptions, performs significantly better than *CCD* and *OD*. Next, we describe the simple association rules that we are going to study and compare.

V. SIMPLE USER ASSOCIATION RULES

In practical cellular systems, users arrive in the network, stay for a while, and depart the network. Such systems would work optimally if we are able to compute the optimal RAIM parameter (if any) and associate and re-associate users optimally whenever a new user arrives, or a user moves or departs the system, or the channel gains change significantly. Such heavy computations are difficult to do online and changing the RAIM parameter and re-associating a large number of users frequently might degrade the system's performance and result in oscillations. To avoid such possible problems, simple association rules have been used in homogeneous cellular systems and proposed in the literature for heterogeneous systems. These rules typically associate users based on physical layer parameters without considering other system's issues such as load balancing among BSs. In this paper, we study some of those rules and propose a new user association rule that we call *Picocell First*. A description of these rules follows:

1) **Received Signal Power**: A user i associates with BS $j^\star$ that provides the highest downlink received signal power, i.e., $j^\star = \arg\max_{j\in\mathcal{B}} \{P_j\ G_{ij}\}$ where P_j and G_{ij} denote the transmit power of BS j and the channel gain between user i and j, on each sub-channel respectively. This association rule has been used in conventional cellular networks. We call it "Current Practice" (CP).
2) **Range Extension (RE)** [16]: A user i associates with BS $j^\star = \arg\min_{j\in\mathcal{B}} \{\delta_{ij}\}$ where δ_{ij} is the path loss from BS j to user i.
3) **Picocell First (PicoF)** [10]: A user i associates with pico BS $j^\star = \arg\max_{j\in\mathcal{B}'} \{\gamma_{ij}\}$ as long as $\gamma_{ij^\star} > \beta$ where β is a tuning parameter. Note that γ_{ij} denotes user i's SINR on each sub-channel. If $\max_{j\in\mathcal{B}'} \{\gamma_{ij}\} < \beta$, user i associates with the macro BS. This rule associates users with pico BSs regardless of their received power from the macro BS as long as the best SINR seen from a pico BS is larger than β. The motivation behind this rule is to bring BSs closer to users and offload data traffic via pico BSs.

For each of these rules, once the physical layer parameters are known, we can compute the values of x_{ij} for all users i and BSs j. To compute the physical layer parameters, we need to fix the resource allocation scheme and its parameters if any, i.e., K, u, $p(u)$ for *OD* and *PSD*. Therefore, for *OD* and *PSD*, to compute the system's performance when the user association is given by a simple association rule, we need to fix K, u, $p(u)$ and to compute the system's performance corresponding to these parameters, and then iterate on these parameters. Note that for *CCD* the resource allocation parameters are fixed. Thus, given a user association $\{x_{ij}\}$, we can compute the solution to $\mathbf{P}^{\ell}_{\mathbf{CCD}}$ by calculating $\sum_i \log(\lambda_i)$.

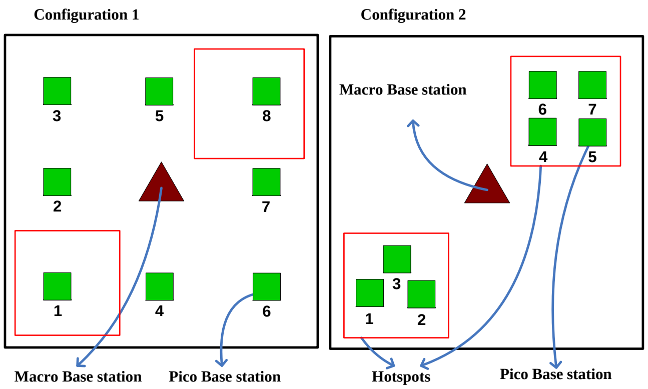

Fig. 2. Picocell locations for Configuration 1 and Configuration 2. The triangle is the macro BS and the squares are the pico BSs. In each configuration, there are two hotspots shown as inner squares.

We now have a unified framework, i.e., the proposed joint user association and resource allocation problems, and we can compute upper bounds on the objective function of the proposed problems. Using this framework, we can compute the optimal resource allocation parameters and the performance metrics (i.e., $\sum_i \log(\lambda_i)$, $\sum_i \lambda_i$, $\min_i \{\lambda_i\}$) when an association rule is given. Note that when we fix the association rule, we can generate a feasible integral solution to each problem $\mathbf{P_{CCD}}$, $\mathbf{P_{PSD}}$, and $\mathbf{P_{OD}}$. If we can find a simple association that yield an objective function close to the corresponding upper bound, then we would have validated the tightness of our bound and the goodness of that simple association rule. Next, we explore the performance of existing and proposed user association and RAIM schemes.

VI. NUMERICAL RESULTS

A. Parameter Settings

We consider a square area of length $L = 500\ m$ that is covered by one macro BS (cell 0) and X pico BSs (cells $j = 1, \cdots, X$). We study two different configurations (see Fig. 2). In configuration 1, eight pico BSs are located inside the square on a 3×3 grid, and in configuration 2, seven pico BSs are located in the square as shown in the figure. There are $N = 60$ users distributed in the system area. To consider the case where users are clustered in some areas in the system, for each configuration, we consider two types of user distribution: uniform (UD) and non-uniform (NUD). With the uniform user distribution, the 60 users are distributed at random uniformly in the square area, while with the non-uniform user distribution, $\frac{2}{3}$ of the users are uniformly distributed at random in the hotspot areas shown in Fig. 2 while the rest is distributed uniformly in the square area. Each hotspot is a square of 170m in length.

The physical layer parameters are based on the 3GPP evaluation methodology document [17] used for Hetnets in

TABLE III
MODULATION AND CODING SCHEMES-LTE

SINR thresholds (in dB)	-6.5	-4	-2.6	-1	1	3	6.6	10	11.4	11.8	13	13.8	15.6	16.8	17.6
Efficiency (in bits/symbol)	0.15	0.23	0.38	0.60	0.88	1.18	1.48	1.91	2.41	2.73	3.32	3.90	4.52	5.12	5.55

TABLE I
PHYSICAL LAYER PARAMETERS

Noise Power	$-174\ \frac{\text{dBm}}{\text{Hz}}$	T_{subframe}	1 ms
P_{pico}	30 dBm	P_{macro}	46 dBm
UE Ant. Gain	0 dB	Sub-channel Bandwidth	180 KHz
Shadowing s.d.	8 dB	User Noise Figure	9 dB
Penetration Loss	20 dB	M (Number of sub-channels)	100
Macro Ant. Gain	15 dBi	Pico Ant. Gain	5 dBi
SC_{ofdm}	12	SY_{ofdm}	14
Path Loss Pico	$140.7 + 36.7\ \log_{10}(d/1000),\ d \geq 10m$		
Path Loss Macro	$128 + 37.6\ \log_{10}(d/1000),\ d \geq 35m$		

TABLE II
REUSE FACTORS AND CORRESPONDING REUSE PATTERNS USED FOR THE CONFIGURATIONS IN FIGURE 2

	Co-channel pico BSs	
reuse factor	Configuration 1	Configuration 2
2	$\{1,3,5,6\},\{2,4,7\}$	$\{1,3,6,8\},\{2,4,5,7\}$
3	$\{1,4,7\},\{2,5\},\{3,6\}$	$\{2,5,6\},\{1,8\},\{3,4,7\}$

LTE. These parameters are shown in Table I. In particular, we use $M = 100$ sub-channels. We use the SINR model introduced in Section III-C that accounts for path loss and slow fading. Slow fading is modeled by a log-normal shadowing with standard deviation of 8 dB, and path losses for pico and macro BSs are given in Table I. The set of reuse factors is $\mathcal{U} = \{1, 2, 3\}$. The respective reuse patterns considered for configuration 1 and configuration 2 are shown in Table II[2]. For each configuration and reuse factor, the set of BSs that use the same set of sub-channels are shown in this table. For example, in configuration 1, when the system uses $u = 2$, the set of sub-channels is divided into two equal and disjoint sets. All pico BSs in the set $\{1, 3, 5, 6\}$ use the first set of sub-channels, and the other pico BSs (i.e., $\{2, 4, 7\}$) use the second set of sub-channels. We assume that the system uses adaptive modulation and coding with discrete rates. Table III taken from [18] and [19] gives us the mapping between the SINR and the efficiency (in bits/symbol) for the modulation and coding schemes (MCS) for LTE. The bit rate obtained by a user that has a SINR between level ℓ and level $\ell + 1$ is $r = \frac{\text{SC}_{\text{ofdm}}\ \text{SY}_{\text{ofdm}}}{T_{\text{subframe}}} e_\ell$ where e_ℓ is the efficiency (bits/symbol) of the corresponding level ℓ, SC_{ofdm} is the number of data subcarriers per sub-channel bandwidth, SY_{ofdm} is the number of OFDM symbols per subframe, and T_{subframe} is the subframe duration in time units. The value of these parameters are shown in Table I.

Our comparisons are based on the following performance metrics:

- GM := Geometric mean rate of the users, i.e., $\sqrt[N]{\prod_{i=1}^{N} \lambda_i}$ (note that for fixed N, maximizing the GM is equivalent to maximizing our objective function);
- Min Rate := Minimum rate among all users, i.e., $\min_i\{\lambda_i\}$;
- TT := Total throughput of the system, i.e., $\sum_{i=1}^{N} \lambda_i$.

"Picocell First" has a tuning parameter β . We assume that β can take any one of the SINR threshold values of the deployed MCS shown in Table III. In the numerical results, we select the value of β that gives the highest possible geometric mean throughput. Two different system configurations and two types of user distribution provide us four scenarios to compare the performance of different combinations of resource allocation and user association schemes. For each scenario, we compute the upper bound for 100 realizations. A realization corresponds to the random placement of the N users in the system area based on the scenario. We have observed that the relative performance of a given rule (or a given RAIM scheme) is the same over all realizations and hence we chose one of the realizations to show the trends.

B. Comparison Results

Tables IV-V provide the results for two typical realizations corresponding to uniform and non-uniform user distributions, respectively. In the row entitled "GM relaxation" in these tables, the upper bounds of the joint user association and resource allocation for *CCD*, *OD*, and *PSD* are provided when all system parameters are computed optimally (i.e, for *OD* and *PSD*, we compute the best K and the best u). To check the tightness of these upper bounds, we compare them to the geometric mean rate of the Picocell First association rule (*GM PicoF* in the tables) computed for the best β. For these realizations, we also consider the system without any pico BS (called "No pico" in Tables IV-V) to see how much gain can be achieved by deploying pico BSs. The results show that:

- *PSD* and *OD* work significantly better than *CCD* in all cases. *PSD* and *OD* perform almost the same with a slight advantage for *PSD*. For *PSD*, we saw gains (with respect to *CCD*) in total throughput in the range of 48% to 101%, and gains in geometric mean rate in the range of 38% to 86% over our 100 realizations per scenario. This is maybe not so surprising under our assumption that the BSs are not coordinated and transmit at all time on all channels allocated to them at full power.
- The association rule *Picocell First* is almost optimal since the geometric mean rate of the Picocell First is very close to the upper bound for *CCD*, *OD*, and for *PSD* when all system parameters, including β, are chosen optimally. This has two consequences. It validates our relaxation approach because an integer feasible solution to the

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[2] Note that in the case of *PSD*, the macro BS is transmitting on all the K sub-channels irrespective of the value of u. Hence, the macro BS is an interferer for all pico BSs. This is not shown in Table II.

TABLE IV

RESULTS FOR CONFIGURATION 1, $u = 1$, AND $P_{pico} = 30\text{dBm}$ AND BEST β

Uniform user distribution

RAIM scheme	CCD	OD	PSD
GM relaxation	1.2925e+6	2.6909e+6	2.9847e+6
GM PicoF	1.2925e+6	2.6535e+6	2.8958e+6
GM No Pico	1.2138e+6	1.2138e+6	1.2138e+6
Min Rate PicoF	0.3986e+6	1.1722e+6	1.2210e+6
Min Rate No Pico	0.8580e+6	0.8580e+6	0.8580e+6
TT PicoF	1.0060e+8	2.1434e+8	2.3814e+8
TT No Pico	0.7290e+8	0.7290e+8	0.7290e+8

Non-uniform user distribution

RAIM scheme	CCD	OD	PSD
GM relaxation	1.0276e+6	2.3465e+6	2.4914e+6
GM PicoF	1.0276e+6	2.1988e+6	2.2707e+6
GM No Pico	1.2210e+6	1.2210e+6	1.2210e+6
Min Rate PicoF	0.3617e+6	0.7139e+6	0.8177e+6
Min Rate No Pico	1.2210e+6	1.2210e+6	1.2210e+6
TT PicoF	0.8630e+8	1.5787e+8	1.9248e+8
TT No Pico	0.7326e+8	0.7326e+8	0.7326e+8

TABLE V

RESULTS FOR CONFIGURATION 2, $u = 1$, AND $P_{pico} = 30\text{dBm}$ AND BEST β

Uniform user distribution

RAIM scheme	CCD	OD	PSD
GM relaxation	1.5742e+6	2.0286e+6	2.1035e+6
GM PicoF	1.5742e+6	2.0170e+6	2.0861e+6
GM No Pico	1.2138e+6	1.2138e+6	1.2138e+6
Min Rate PicoF	0.9900e+6	0.9625e+6	0.9459e+6
Min Rate No Pico	0.8580e+6	0.8580e+6	0.8580e+6
TT PicoF	1.0534e+8	1.6460e+8	1.5691e+8
TT No Pico	0.7290e+8	0.7290e+8	0.7290e+8

Non-uniform user distribution

RAIM scheme	CCD	OD	PSD
GM relaxation	1.7107e+6	3.0730e+6	3.2641e+6
GM PicoF	1.6459e+6	3.0422e+6	3.2274e+6
GM No Pico	1.2210e+6	1.2210e+6	1.2210e+6
Min Rate PicoF	0.4950e+6	0.9504e+6	1.0384e+6
Min Rate No Pico	1.2210e+6	1.2210e+6	1.2210e+6
TT PicoF	1.0953e+8	2.3760e+8	2.7012e+8
TT No Pico	0.7326e+8	0.7326e+8	0.7326e+8

proposed problems achieves almost the same geometric mean rate as the solution of the relaxed problem. It also shows that *Picocell First* is a very good yet simple association rule. A similar results was obtained in [10] for a different framework. Since it is near optimal, we will use *Picocell First* when we want to compare the resource allocation schemes in term of minimum rate and total throughput.

- The comparison of the system's performance (using *Picocell First*) between the system with and without pico BSs ("No pico" in the tables) shows that pico BSs can significantly increase the performance of the system. We saw gains (with respect to the system without pico BSs) in total throughput in the range of 136% to 231%, and gains in geometric mean rate in the range of 73% to 145% over our 100 realizations per scenario.

C. In depth study of PSD

We now study *Partially Shared deployment* in more details (we have performed the same study for *OD* and found similar results). As mentioned in Sections III-IV, reuse factor u and reuse pattern $p(u)$ are two variables for $\mathbf{P_{PSD}}$. The performance of the system in terms of the geometric mean rate as a function of u for the given set of reuse patterns in Table II and for different scenarios is shown in Fig. 3. For all scenarios, $u = 1$ is the best reuse factor. We believe that this is due to the fact that we are optimizing the system's performance using two degrees of freedom, i.e., over the users' association parameters $\{x_{ij}\}$ and the channel allocation parameter K , which allows us to be "aggressive" in terms of reuse factor. The results show that mitigating interference among pico BSs by choosing a higher reuse factor u comes at the expense of performance when the system's parameters are chosen optimally.

In the following, we fix $u = 1$, and we compare the performance of the simple association rules with the upper bound as a function of K. For each value of K, we compute the upper bound, i.e., the solution to the relaxed problem $\mathbf{P'}^{\ell}_{\mathbf{PSD}}$, and the corresponding geometric mean rate for each association rule. The results for the 4 scenarios are shown in Figures 4 to 7 which all show the same relative performances. The curve corresponding to the upper bound is labeled *UP* in the figures. Since *CCD* is often considered as the preferred option in Hetnets, we also show the upper bound for *CCD* for each scenario. The comparison between the upper bounds for *PSD* and *CCD* for the 4 scenarios, shows that *PSD* performs better than *CCD* for a large range of K, i.e., that even if the operator cannot choose K optimally, he should still prefer *PSD* over *CCD* under our assumptions. Figures 4 to 7 also show that the optimal value of the channel allocation parameter K is highly dependent on the deployed association and on the scenario at hand.

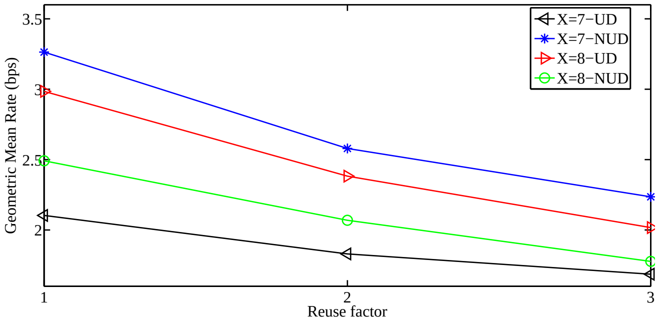

Fig. 3. PSD: Geometric mean rate as a function of u for different configurations and different user distributions

The comparison between the geometric mean rate of the simple association rules and the upper bound for *PSD* shows that "Picocell First" almost always performs the best of the three rules and that for a range of values of K. The results also show that "Current Practice" does not perform well in

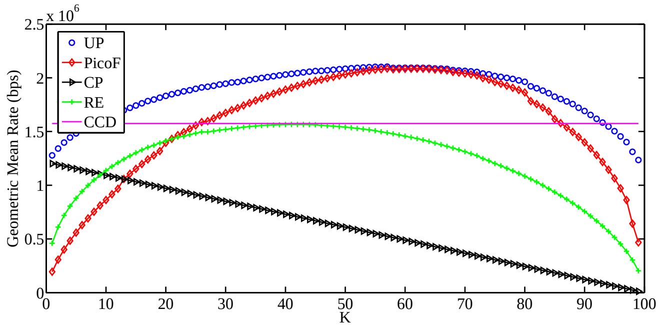

Fig. 4. PSD: configuration 2, and UD: Geometric mean rate as a function of K for $u = 1$

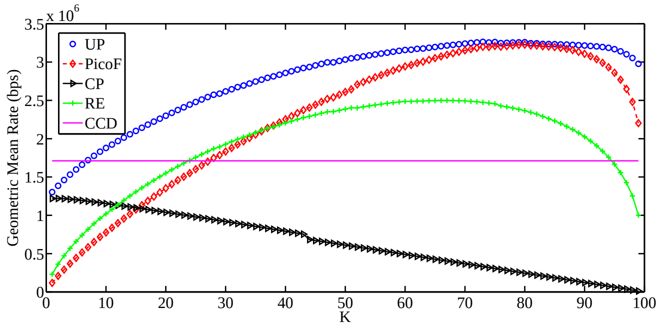

Fig. 5. PSD: configuration 2, and NUD: Geometric mean rate as a function of K for $u = 1$

any of the scenarios. When K is not chosen optimally, the performance of "Picocell First" can be far from the upper bound. We believe that this can be explained by the fact that if resource allocation is not performed well, load balancing becomes a major issue and none of our simple association rules take load balancing into account.

VII. CONCLUSIONS

We have studied the problem of joint user association and resource allocation in Hetnets that consist of a macrocell and some picocells. We have considered three channel allocation schemes, and assumed that all the BSs are transmitting all the time on all their allocated channels. The proposed problems are non-convex integer programs, and hence it is impossible to efficiently obtain exact solutions. We have, therefore, developed techniques to obtain upper bounds on the system's performance. Numerical results show that the proposed upper bounds are tight and can be used as benchmarks to quantify how well different user association rules and resource allocation schemes perform.

Our numerical results indicate that significant performance gains are achievable for Hetnets if the system uses the right combination of user association and resource allocation. Gains in total throughput in the range of 140% to 224%, and gains in geometric mean rate in the range of 75% to 137%, are achievable for Hetnets using pico BSs. *Partially Shared deployment* and *Orthogonal deployment* perform significantly better than *Co-channel deployment*, and selecting an aggressive reuse factor u (i.e., $u = 1$) among pico BSs can lead to

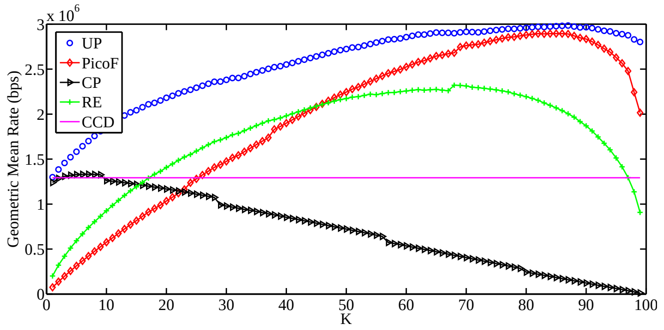

Fig. 6. PSD: configuration 1, and UD: Geometric mean rate as a function of K for $u = 1$

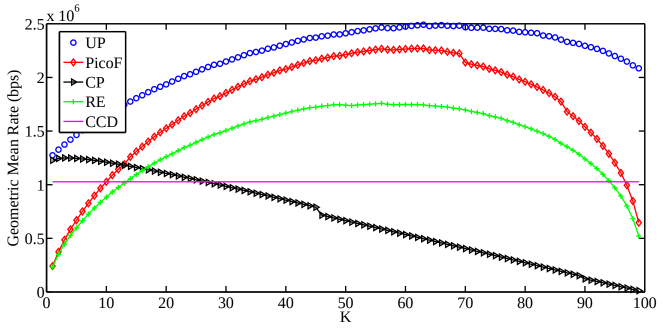

Fig. 7. PSD: configuration 1, and NUD: Geometric mean rate as a function of K for $u = 1$

significant gains. Noting the significant impact of association rules on the performance of Hetnets we have proposed a new user association rule. Our results show that rules which favor associating users with pico BSs (e.g. "Picocell First" and "Range Extension") yield *significantly better* performance than the conventional association rule if their tuning parameters are chosen properly and if the RAIM parameters have been optimally chosen. Because of the flat channel assumption used in our proposed static framework, some of the advantages of LTE such as channel dependent scheduling could not be evaluated. A substantial amount of work still remains to be performed to remove some assumptions made in our analysis.

APPENDIX A
PROOF OF THEOREM 1

Let $\mathcal{H}'$ and $\mathcal{H}'_\ell$ denote the set of optimal solutions for $\mathbf{P'_{OD}}$ and $\mathbf{P'^{\ell}_{OD}}$, respectively. The following claim shows that the set of exact solutions to $\mathbf{P'^{\ell}_{OD}}$ is a subset of the set of exact solutions to $\mathbf{P'_{OD}}$ so that solving $\mathbf{P'^{\ell}_{OD}}$ is equivalent to solving $\mathbf{P'_{OD}}$, and vice versa.

Claim 1. *Given problems $\boldsymbol{P'_{OD}}$ and $\boldsymbol{P'^{\ell}_{OD}}$, we have:*

$$\begin{aligned}\mathcal{H}' \;=\; \Bigg\{&(\{x_{ij}\},\{\alpha_{ij}\}) \mid \; x_{ij}\alpha_{ij} = \frac{x_{ij}}{\sum_{i\in\mathcal{N}} x_{ij}} \\ &\{x_{ij}\} \in \mathcal{H}'_\ell,\ \forall i \in \mathcal{N},\ \forall j \in \mathcal{B}\Bigg\}\ . \qquad (6)\end{aligned}$$

Proof : Let us assume $(\{x_{ij}\},\{\alpha_{ij}\}) \in \mathcal{H}'$, and there exists some $i_0 \in \mathcal{N}$ and $j_0 \in \mathcal{B}$ for which $\alpha_{i_0 j_0} \neq \frac{1}{\sum_{i\in\mathcal{N}} x_{ij_0}}$ while $x_{i_0 j_0} = 1$. For such i's, let us define $\mathcal{U}(j_0) =$

$\{i \in \mathcal{N} \mid x_{ij_0} = 1\}$ If ${\alpha'}_{ij_0} = \frac{1}{\sum_{i \in \mathcal{N}} x_{ij_0}}$ for all $i \in \mathcal{U}\,(j_0)$ and ${\alpha'}_{ij} = \alpha_{ij}$ for all $i \in \mathcal{N}$ and $j \in \mathcal{B}$ ($i \notin \mathcal{U}\,(j_0)$ and $j \neq j_0$), then $(\{x_{ij}\}, \{{\alpha'}_{ij}\})$ is feasible for $\mathbf{P'_{OD}}$, and according to Lemma 1: $\sum_{i \in \mathcal{U}(j_0)} \log({\lambda'}_i) > \sum_{i \in \mathcal{U}(j_0)} \log(\lambda_i)$ where ${\lambda'}_i$ and λ_i are the user i's rate corresponding to the scheduling coefficients ${\alpha'}_{ij_0}$ and α_{ij_0}, respectively. Hence, there exits another feasible solution $(\{x_{ij}\}, \{{\alpha'}_{ij}\})$ that achieves a larger objective value than $(\{x_{ij}\}, \{\alpha_{ij}\})$. This contradicts the assumption that $(\{x_{ij}\}, \{\alpha_{ij}\}) \in \mathcal{H}'$. The inverse can be proved by using Lemma 1 and following the same argument as above. $\square$

Therefore, there exists an onto mapping between the elements of ${\mathcal{H}'}_\ell$ and $\mathcal{H}'$ so that an exact solution $(\{x_{ij}\}, \{\alpha_{ij}\})$ to $\mathbf{P'_{OD}}$ corresponds to an exact solution $(\{x_{ij}\})$ to $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ with scheduling coefficients $\frac{x_{ij}}{\sum_{i \in \mathcal{N}} x_{ij}}$, and vice versa. This mapping is not a one-to-one mapping since in some solutions of $\mathbf{P'_{OD}}$ there might exist some $i_0 \in \mathcal{N}$ and $j_0 \in \mathcal{B}$ for which $\alpha_{i_0 j_0} > 0$ while $x_{i_0 j_0} = 0$. Note that this does not change users' throughput in $\mathbf{P'_{OD}}$ since $\alpha_{i_0 j_0} x_{i_0 j_0} = 0$. Based on Claim 1 and the structure of problems $\mathbf{P'_{OD}}$ and $\mathbf{P'}^{\ell}_{\mathbf{OD}}$, it can be verified that the optimal solutions to $\mathbf{P'_{OD}}$ and $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ result in the same users' throughput. Hence, $\mathbf{P'_{OD}}$ and $\mathbf{P'}^{\ell}_{\mathbf{OD}}$ are equivalent problems. This completes the proof. $\square$

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