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Technical report

Providing Consistent Rates for Backhauling of Mobile Base Stations in Public Urban Transportation

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Abstract

We consider a scenario in which an operator installs (small cell) base stations on top of city buses to offer better quality of experience (QoE) to their passengers. In that case, providing a consistent backhaul rate (i.e., a constant rate at all time) to these base stations could help mitigate the effects of mobility on the QoE. Specifically, we perform the analysis to determine the maximum consistent backhaul rate that can be offered to a bus on a given route, given the resource allocated by the operator to backhauling, by taking advantage of the fact that different buses on that route will see different conditions at a given time. We also consider the case where we allow a small outage probability, i.e., that the consistent rate is not provided for a small proportion of time. We show that by allowing an outage probability of only 1% we can increase the achievable backhaul rate by 50%. We then show how to compute the Pareto frontier (rate region) of the achievable consistent backhaul rates when there are two bus routes. The analysis is performed under an independence assumption and hence we validate our results by simulations. Altogether, the cost of consistency is very high, but it can be partly mitigated by allocating the unused backhaul capacity to best effort services in real time.

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1 Introduction

Nowadays, many public urban transportation systems provide or are considering providing Internet access to their passengers. To accomplish that, they should be furnished with small cell base stations (SCB) [1]. SCBs are connected to the Internet via macro base stations (BTS). The radio link between an SCB and its current BTS is called a *backhaul link*. The BTS has to allocate part of its resources (i.e., part of its subchannels) to these backhaul links and this affects the Quality of Experience (QoE) seen by the passengers.

We consider a scenario in which an operator installs (small cell) base stations on top of city buses to offer better QoE to their passengers. In that case, providing a consistent backhaul rate (i.e., a constant rate at all time) to these base stations could help mitigate the effects of mobility on the QoE. Specifically, we explore the challenges and solutions to provide a maximum consistent backhaul rate to buses *everywhere at any time* in spite of their mobility, the possibly varying (but bounded) number of buses in the cell, and their time-varying channels. We will provide the analysis for the case of one bus route first followed by the case with two different bus routes. In large cities, there are typically more than one bus per line[1] present in a cell coverage area at the same time. As the buses of one line follow the same path, they have similar mobility patterns, and hence we expect them to have the same channel characteristics in distribution.

The interesting research questions that arise related to the problem of offering constant backhaul rates to lines of buses within a cell are:

- What is the maximum consistent rate that can be offered simultaneously to buses of a single line given that the resources dedicated to this type of backhauling by the BTS are known?
- What are the maximum consistent rates that can be offered simultaneously to buses of different lines given that the resources dedicated to this type of backhauling by the BTS are known?
- If the operator is willing to accept a small outage probability (i.e., not comply with the backhaul consistent rate guarantee for a small percentage of the time), what is the gain in the backhaul rates that can be offered?

We answer all these questions and in particular, we provide the Pareto frontier of the backhaul rates that can be offered to two lines of buses in the general case. We propose models that can capture both a deterministic or a random number of buses present in the cell. These models are used to determine the possibly different consistent backhaul rates that can be offered to the buses in the two lines, given the resources dedicated to this type of backhauling by the BTS. Our contributions are summarized below:

[1] We will use the term line and route interchangeably.

- We derive the maximum consistent backhaul rate that can be provided in a cell, to each bus of one route, both for deterministic and random number of buses, for a given amount of resources dedicated by the BTS to bus backhauling, for a given outage probability under the "independence" assumption that the rate distributions seen by each bus in the route are i.i.d. and memoryless.

- We determine the Pareto frontier of the consistent backhaul rates (consistent within a bus route) that can be offered to buses on 2 bus routes under the same assumption for a given amount of resources dedicated by the BTS to bus backhauling and for a given outage probability.

- We validate our approach by simulations and obtain several engineering insights:

  - Our independence assumption yields results that are very close to those obtained by simulation where this assumption does not hold.
  - We show that by allowing a very small outage probability, the backhaul consistent rate for each bus can be increased significantly for the same amount of available resources.
  - We show that the cost of consistency is very high and this can be partly mitigated by allowing the resources dedicated to backhauling to be used on a best effort basis by other users of the BTS.

This report is organized as follows. In Section 2 we discuss some related work. The model and analysis are presented in Section 3. In Section 4, we use our analytical results to provide numerical results and some engineering insights. Finally, we conclude our work in Section 5.

2 Related Work

There has been a significant amount of research in the area of vehicular networking [2, 3, 4]. In [5], the authors investigate how rate predictions of mobile users can be utilized to improve long-term fairness. They use the α-fair criteria to formulate a predictive long-term resource allocator that improves fair user service over multiple cells. Throughput optimization and achieving fairness are the goals of [6]. It considers the association problem, but for WiFi APs.

Wireless backhauling for 4G was the focus of [7]. In order to improve the performance, the authors propose a mesh backhaul network. A vehicular backhaul is presented in [8], where a centralized software defined based architecture is proposed. This architecture consists of a central controller that acts as a service broker.

In most of the works related to backhauling for either vehicular networks or any other wireless network, the authors propose architectures that offer some improvements in terms of one of the metrics of interest, together with procedures for device association, connectivity and mobility management. Nevertheless, in most of them there is no analysis, and all the outcomes are based on experimental or simulation results.

The works somehow related to ours that provide some analysis on networks created from vehicular users are [9] and [10]. In [9], an interference-based analysis is performed in order to obtain the worst-case capacity in a Vehicular Ad-hoc Network (VANET). A very detailed analysis based on a very realistic 802.11 communications model is performed in [10]. That model evaluates the throughput performance of multiple vehicles that share the available wireless resources (from the AP). It captures the effect of road capacity, vehicle density, and different velocities of the users. Nevertheless, the proposed approach does not guarantee any consistent data rate at any time, as opposed to the approach we follow here where we are able to guarantee a constant rate to small cells.

Other works related to ours in some ways are [11] and [12]. These focus on cellular-based vehicular networks but address different research questions. In [11], the authors focus on frequency allocation and power control. They propose a regular frequency reuse pattern and calculate users' achievable capacity. They also propose an adaptive power control algorithm to reduce co-channel interference. However, there is no performance guarantee for the users. In [12], the vehicular mobility performance for a 5G cooperative MIMO small cell network is analyzed. The authors also consider the co-channel interference. The analysis is based on stochastic geometry. While the analysis is quite involved and in depth, it does not capture the case when we want to offer a given QoS per user, and they do not consider backhauling neither.

Summarizing, we found no work that addresses analytically the backhauling of small cells furnishing city buses, and especially not in the context of offering them consistent rates.

3 Performance modeling and analysis

3.1 The system

We consider buses furnished with their own small cell base stations (SCBs) within the coverage region of a macro BTS. The link between the bus SCB and the BTS is the backhaul link. We focus on the downlink in this paper, i.e., from the BTS to the SCBs.

Due to their mobility and time varying channels, the buses will see a time varying per-channel data rate (a function of the modulation and coding (MCS) scheme and the per-channel Signal to Interference and Noise Ratio (SINR)). To capture these effects, we model the per-channel data rate of a bus in route i as a random variable, R_i. It is clearly a function of time, but we omit this in our notations for simplicity. We assume that all buses in a given route see the same rate distribution. Specifically, we assume flat channels at a given time. We also assume that the adaptive modulation and coding scheme (MCS) that is used to translate the SINR into a per-channel rate is discrete with m possible values (typically $m = 15$) [13, 14]. More details on this are given in Section 4. Hence, R_i is a discrete random variable with values $\{r_1, r_2, \ldots, r_m\}$, such that $r_1 < r_2 < \ldots < r_m$, and with a cumulative distribution function $F_{R_i}(x)$. The probability $p_{i,k}$ that a bus in route i will have a given per-channel rate r_k depends on the SINR it sees and hence on the bus mobility pattern and the route environment. This means that the probabilities $p_{i,k}$ are different for different routes.

In this paper, we assume that the set of probabilities, $\{p_{1,k}, p_{2,k}, \ldots, p_{m,k}\}$ is given (it can easily be obtained from measurements). This is what we call the rate distribution in the following. We also assume that the time is slotted and the rates seen by a bus in consecutive time slots are independent (i.e., they are memoryless), and the per-channel rates of different buses in the same route are i.i.d. These are strong assumptions. However, in Section 4, we relax these assumptions and validate the theoretical results with realistic simulations based on assumptions that depart from the theoretical ones.

We consider a case with one line of buses and a case with two lines of buses. The number of lines can be generalized to more than two, but visually it is more difficult to illustrate the performance. We provide a discussion on a higher number of routes at the end of this section.

System model: We consider the downlink of a single macrocell and assume full coverage all the time. The BTS allocates a set of K channels to bus-backhauling. Its transmission power budget on these K channels is P_T.[2] We assume that the BTS power is allocated equally among all the channels, i.e., the transmission power per channel is $\frac{P_T}{K}$. We consider a time-slotted system where a time slot corresponds to a frame duration (e.g., 10 ms), and we assume that during a time slot the channel characteristics do not change, but channel gains can vary from one time slot to another.

Number of buses: We perform our analysis for two scenarios. In the first scenario, we assume that the number of buses on line i in the cell is constant and equal to n_i. In the second one, we perform the analysis for a more realistic case where buses come into and go out of the covered region. We model the number of buses on line i in the cell (in a given time slot)[3] with the random variable N_i that takes its values in $\{0, 1, \ldots, N_{i,max}\}$ with the probability mass functions (PMF) $\alpha_{i,j}$, $j = 0, 1, \ldots, N_{i,max}$. We assume that the per-line PMF is given (it can easily be obtained from measurements).

In the following, we first consider the case with one bus line and a constant number of buses in the cell, then the case with one bus line and a varying number of buses in the cell before considering the case of 2 bus lines.

[2]Note that the BTS is in fact dedicating two types of resources to bus-backhauling, namely channels and power budget.

[3]We assume that the number of buses in the next time slot will change randomly according to the corresponding PMF.

3.2 The case of one bus line and a constant number of buses in the cell

We consider the case where there is one bus line and there are n buses at all time in the cell. As already mentioned in Section 1, we are interested to provide each bus in a given route the same *consistent* backhaul rate, U (consistent meaning that the rate remains constant and is delivered at all time). We strive to assign the buses a rate as high as possible given the constrained resources (finite number of channels and limited power). We might be interested in providing a larger rate to these buses by trading off the "at all time" requirement. Hence, we consider the case where we have to deliver the constant rate U to a bus at least $1-\varepsilon$ of the time with ε being less than a few percents. We call ε the *outage probability*.

We assume that the BTS allocates its resources on the downlink using a scheduler as in [15]. Specifically, a bus j receives all the K channels for a time duration of t_j[4]. As buses see variable per-channel rates, in order to provide a given backhaul rate U, a different portion of time needs to be allocated to each bus in every time slot. Specifically, a bus might have a very bad channel at a given time (leading to poor per-channel rate) and in that case, more time needs to be allocated to that bus in order to maintain the desired data rate. On the other hand, when the bus has a good channel, less time needs to be assigned to achieve the consistent rate.

In order to offer the data rate U to bus j that sees a per-channel rate r in a given time slot t, $x_j(t)$ (the ratio of time to get all the resources in that time slot) should be equal to $x_j(t) = \frac{U}{K \cdot r}$.

If we want to provide a given U to all n buses with no outage, we should have

$$X_n(t) = \sum_{j=1}^{n} x_j(t) \leq 1. \tag{1}$$

For ease of notations, we discard the time parameter t. We call the random variable X_n the *resource utilization ratio*. Since all the buses belong to the same route, they see the same distribution for their per-channel data rate, R. The resource utilization ratio is then, under our assumptions, the sum of n i.i.d. random variables $\frac{U}{KR}$.

If we want to offer U to all buses all the time, then $P(X_n > 1)$ should be equal to zero. If we accept an outage probability $\varepsilon > 0$, then

$$P(X_n > 1) \leq \varepsilon, \tag{2}$$

which can be loosely written as

$$P\left(\frac{1}{R} + \ldots + \frac{1}{R} \leq \frac{K}{U}\right) \geq 1 - \varepsilon, \tag{3}$$

where the sum has n terms. The left-hand side of Eq.(3) is the Cumulative Distribution Function (CDF) of the random variable that is the sum of the inverses of the random variable R at point $\frac{K}{U}$. As we know from basic probability theory [16], the

[4]This value is a portion of the total duration of the time slot.

CDF of the sum of two random variables, $W = Y + Z$, is equal to the convolution of the CDF of the first variable with the Probability Mass (Density) Function (depending if the random variable Z is discrete or continuous) of the second random variable, i.e.,

$$F_W(w) = F_Y(w) * P[Z = w]. \tag{4}$$

As we assume here that the per-channel data rate can take values from a finite set of m values, we have used PMF in Eq.(4). We will do that in the following as well. Further, we can write the CDF of the sum $Y = \frac{1}{R} + \ldots + \frac{1}{R}$ as

$$F_Y(x) = P\left[\frac{1}{R} \leq x\right] * P\left[\frac{1}{R} = x\right] * \ldots * P\left[\frac{1}{R} = x\right], \tag{5}$$

where $*$ denotes the convolution operation.

Our objective is to find the maximum value of U_{max} that does not violate Eq.(2), given K, n, ε, the distribution of R, and P_T. Next, we will consider closely the characteristics of the CDF and PMF of the random variable $\frac{1}{R}$. For the CDF of $\frac{1}{R}$ we can write

$$P\left[\frac{1}{R} \leq x\right] = P\left[R \geq \frac{1}{x}\right] = f(x) \tag{6}$$

As it can be seen from Eq.(6), the CDF of $\frac{1}{R}$ is, in fact, the Complementary CDF (CCDF) of R, but at point $\frac{1}{x}$. This function is shown in Fig. 1. The other terms of Eq.(5) are the PMF of $1/R$ and can be written as

$$P\left[\frac{1}{R} = x\right] = P\left[R = \frac{1}{x}\right] = \sum_{k=1}^{m} p_k \cdot \delta\left(x - \frac{1}{r_k}\right) = g(x), \tag{7}$$

where δ is the Dirac delta function. See Fig. 2 for a representation of $g(x)$ that is a sum of Dirac delta functions.

Basically, after performing the convolution of the stair-case function $f(x)$ with $g(x)$ once, the new function will again be a stair-case function, although with a higher number of "jumps". As we perform the convolution operation again (up to $n - 1$ times) the resultant function will be smoother, and finally the shape of $F_Y(x)$ will be as in Fig. 3.

The following theorem gives the maximum achievable consistent data rate when there is only one bus route.

Theorem 1. *The maximum consistent data rate that can be guaranteed in a cell to every bus (following the same single route) with a probability* $1 - \varepsilon$*, when there are* K *channels for bus-backhauling and* n *buses in the cell at all time, is*

$$U_{max} = \frac{K}{F_Y^{-1}(1 - \varepsilon)}. \tag{8}$$

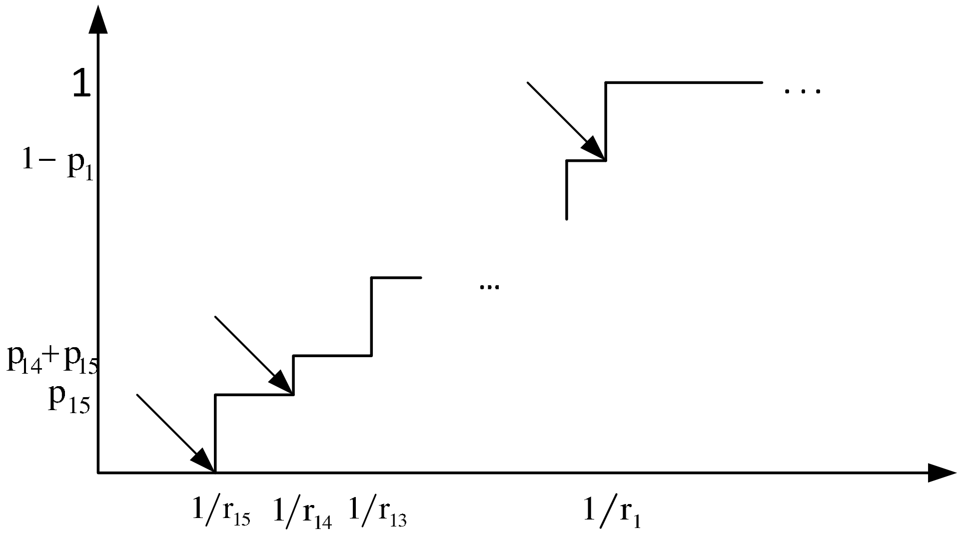

Figure 1: The function $f(x)$.

Proof. We can rewrite the condition given in Eq.(3) as

$$F_Y\left(\frac{K}{U}\right) \geq 1 - \varepsilon. \tag{9}$$

Let θ_0 be such that $F_Y(\theta_0) = 1 - \varepsilon$, i.e., $\theta_0 = F_Y^{-1}(1 - \varepsilon)$ (see Fig. 3). The function F_Y is an increasing function, and the condition in Eq.(9) is fulfilled for $x = \frac{K}{U} \geq \theta_0$. Since $\frac{K}{U}$ decreases as U increases, the maximum value of the consistent rate guaranteed to every bus, for which Eq.(9) still holds, is achieved at the point θ_0. So, the maximum consistent data rate that can be guaranteed $1 - \varepsilon$ of the time to every bus is

$$U_{max} = \frac{K}{\theta_0}. \tag{10}$$

□

Note that θ_0 is a function of ε, n, the distribution of R, and that U_{max} is a function of K, the distribution of R, ε, n, and P_T (the transmission budget allocated to the K channels). If P_T is decreased (resp. increased), the per-channel SINR would decrease (resp. increase), and so would the per-channel rate.

3.3 The case of one bus line and a random number of buses in the cell

While the assumption of constant number of buses in the cell was made as a first step in the analysis, it is not realistic. Namely, different buses will enter and leave the cell coverage at different times. Usually, a single base station will not cover the complete bus route. We consider the case of a single bus route where N_{max} is the maximum number of buses that can be present simultaneously in the cell. The

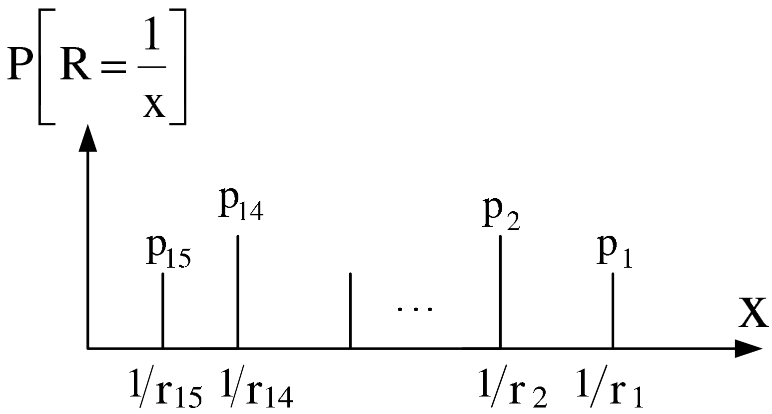

Figure 2: The function $g(x)$.

probability of having j buses in the cell is α_j. The analysis is different from the previous case.

Theorem 2. *The maximum consistent data rate that can be guaranteed in a cell to every bus (following the same single line) with a probability* $1 - \varepsilon$*, when the number of buses N is random, is the solution of the equation*

$$\sum_{j=0}^{N_{max}} F_{Y,j}\left(\frac{K}{U_{max}}\right)\alpha_j = 1 - \varepsilon, \tag{11}$$

where $F_{Y,j}(x)$ *is a generalization of* $F_Y(x)$ *from Eq.(5) to the case where the number of terms is j.*

Proof. For the case of random number of buses simultaneously being present in the cell, Eq.(3) will lead to

$$\sum_{j=0}^{N_{max}} P\left(\underbrace{\frac{1}{R} + \ldots + \frac{1}{R}}_{j} \leq \frac{K}{U}\right) P(N = j) \geq 1 - \varepsilon, \tag{12}$$

where $P(N = j) = \alpha_j$. Let $F_{Y,j}\left(\frac{K}{U}\right) = P\left(\underbrace{\frac{1}{R} + \ldots + \frac{1}{R}}_{j} \leq \frac{K}{U}\right)$. Then, the left-hand side of Eq.(12) can be written as

$$\sum_{j=0}^{N_{max}} F_{Y,j}\left(\frac{K}{U}\right)\alpha_j \geq 1 - \varepsilon \tag{13}$$

It follows that $F_{Y,0}\left(\frac{K}{U}\right) = 1$, $F_{Y,1}\left(\frac{K}{U}\right) = P\left(\frac{1}{R} \leq \frac{K}{U}\right) = f\left(\frac{K}{U}\right)$, and $F_{Y,j}\left(\frac{K}{U}\right)$, for $j \geq 2$ is $F_Y\left(\frac{K}{U}\right)$ with $n = j \geq 2$ terms. The functions $F_{Y,j}$ (except for $j = 0$ when it is a constant) are increasing functions in $\frac{K}{U}$, i.e., they decrease as U increases. The same happens with the sum in Eq.(13). As increasing the value of the left-hand side of Eq.(13) means decreasing of the value of U, the maximum consistent rate is achieved for the smallest possible allowed value of $\sum_{j=0}^{N_{max}} F_{Y,j}\left(\frac{K}{U}\right)\alpha_j$, which is $1-\varepsilon$, i.e.,

$$\sum_{j=0}^{N_{max}} F_{Y,j}\left(\frac{K}{U_{max}}\right)\alpha_j = 1-\varepsilon.$$

□

Note that U_{max} is a function of ε, K, P_T, the distribution of R, and the α_j's. The solution of Eq.(13) can be obtained numerically.

3.4 The case with 2 bus lines and a constant number of buses per line

We now consider two lines of buses, each with a different per-channel data rate distribution. We denote the corresponding random variables by R_1 and R_2, and the number of buses by n_1 and n_2, respectively. Again, we want to assign the same consistent rate to each bus of a given line, say U_1 and U_2. We get the following constraint

$$P\left[\underbrace{\frac{U_1}{KR_1}+\ldots+\frac{U_1}{KR_1}}_{n_1}+\underbrace{\frac{U_2}{KR_2}+\ldots+\frac{U_2}{KR_2}}_{n_2}\leq 1\right]\geq 1-\varepsilon, \tag{14}$$

and obtain the following result:

Result 3. *The rate region for the consistent data rates for n_1 buses of line 1 and n_2 buses of line 2 is given by*

$$P\left[R_1 \geq \frac{U_1}{Kx}\right]*P\left[R_1 = \frac{U_1}{Kx}\right]*\ldots*P\left[R_1 = \frac{U_1}{Kx}\right]*P\left[R_2 = \frac{U_2}{Kx}\right]*\ldots*P\left[R_2 = \frac{U_2}{Kx}\right] \geq 1-\varepsilon. \tag{15}$$

There are clearly many possible pairs (U_1, U_2) that are feasible. We will compute the Pareto frontier that can be obtained as follows. First, we determine the values of $U_{1,max}$ when the data rate assigned to buses of line 2 is 0 (from Eq.(10)). Then, we determine $U_{2,max}$ from Eq.(10), when the data rate assigned to buses of line 1 is 0. This way we have the two end points of the rate region (Pareto frontier). In order to obtain the complete Pareto frontier, for each value of U_1 in the interval $(0, U_{1,max})$, we determine the maximum possible value of $U_{2,max}$ such that the inequality Eq.(15) is not violated. This way we obtain the complete Pareto frontier, as in Fig. 4.

The special case of $\varepsilon = 0$. If we want to guarantee a constant backhaul rate U_1 (resp. U_2) to all the buses of line 1 (resp. line 2) in the cell, then since there is a non-zero probability that all the buses see the lowest per channel rate r_1 we get the following result:

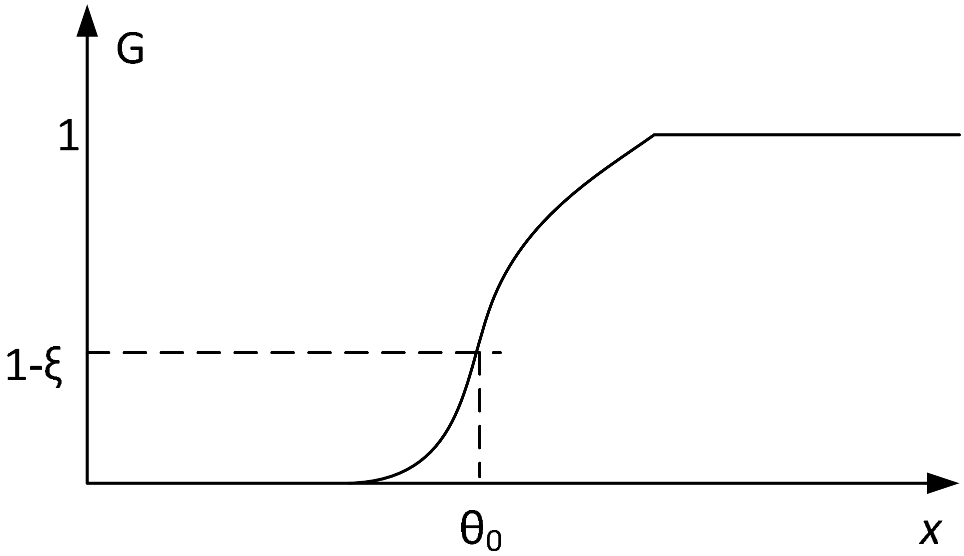

Figure 3: The function $F_Y(x)$.

Result 4. *The rate region for $\varepsilon = 0$ is the line*

$$\frac{n_1 U_1}{K r_1} + \frac{n_2 U_2}{K r_1} = 1. \tag{16}$$

This result is obtained from

$$P\left(\frac{U_1}{KR_1} + \ldots + \frac{U_1}{KR_1} + \frac{U_2}{KR_2} + \ldots + \frac{U_2}{KR_2} > 1\right) = 0,$$

where there are n_1 $\frac{U_1}{KR_1}$ and n_2 $\frac{U_2}{KR_2}$ terms in the previous equation, respectively. This is equivalent to

$$P\left(\frac{U_1}{KR_1} + \ldots + \frac{U_1}{KR_1} + \frac{U_2}{KR_2} + \ldots + \frac{U_2}{KR_2} <= 1\right) = 1,$$

or equivalently

$$\frac{U_1}{KR_1} + \ldots + \frac{U_1}{KR_1} + \frac{U_2}{KR_2} + \ldots + \frac{U_2}{KR_2} <= 1.$$

In the scenario of $\varepsilon = 0$, we must assume that at some time slot all the users will see the lowest per-channel rate, r_1. Since $\varepsilon = 0$, even in that case (when everyone sees r_1) the resources shouldn't be exceeded. Hence, $R_1 = r_1$ and $R_2 = r_2$, after replacing them in Eq.(3.4) and assuming that all the resources are fully utilized (left-hand side of Eq.(3.4) equal to 1), we obtain Eq.(16).

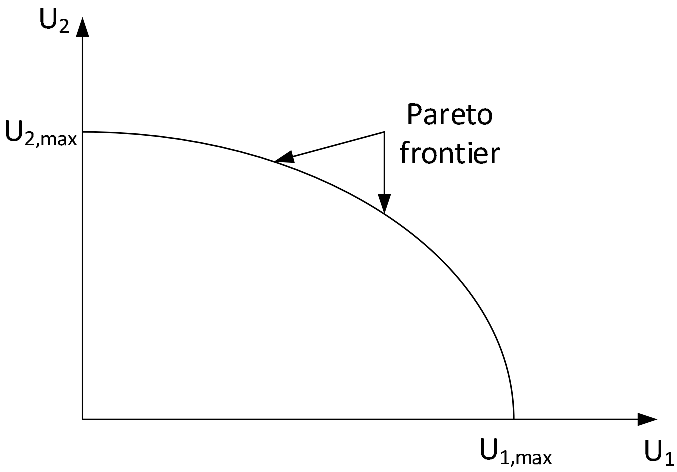

Figure 4: The Pareto frontier.

3.5 The case with 2 bus lines and a varying number of buses per line

Recall that in that case, the maximum number of buses in the system is $N_{1,max}$ for bus line 1 and $N_{2,max}$ for bus line 2. Recall that the probability of having j_1 buses of line 1 present in the system is $P[N_1 = j_1] = \alpha_{1,j_1}$ (for $j_1 \in \{0, 1, \ldots, N_{1,max}\}$). The corresponding probability for line 2 is $P[N_2 = j_2] = \alpha_{2,j_2}$, for $j_2 \in \{0, 1, \ldots, N_{2,max}\}$. We assume that these PMFs are given. Recall the constraint

$$P\left[\underbrace{\frac{U_1}{KR_1} + \ldots + \frac{U_1}{KR_1}}_{N_1} + \underbrace{\frac{U_2}{KR_2} + \ldots + \frac{U_2}{KR_2}}_{N_2} \leq 1\right] \geq 1 - \varepsilon, \tag{17}$$

which gives

$$\sum_{j_1}\sum_{j_2} P\left[\underbrace{\frac{U_1}{KR_1} + \cdots + \frac{U_1}{KR_1}}_{N_1} + \underbrace{\frac{U_2}{KR_2} + \cdots + \frac{U_2}{KR_2}}_{N_2} \leq 1 | N_1 = j_1, N_2 = j_2\right]$$
$$\cdot P[N_1 = j_1, N_2 = j_2] \geq 1 - \varepsilon. \tag{18}$$

The number of line 1 buses present in the system is independent of the number of line 2 buses. So,

$$P[N_1 = j_1, N_2 = j_2] = P[N_1 = j_1]P[N_2 = j_2] = \alpha_{1,j_1}\alpha_{2,j_2},$$

and the left-hand side of the above inequality yields

$$\sum_{j_1=0}^{N_{1,max}}\sum_{j_2=0}^{N_{2,max}} P\left[\underbrace{\frac{U_1}{KR_1} + \ldots + \frac{U_1}{KR_1}}_{j_1} + \underbrace{\frac{U_2}{KR_2} + \ldots + \frac{U_2}{KR_2}}_{j_2} \leq 1\right]\alpha_{1,j_1}\alpha_{2,j_2}.$$

In the previous equation, for the trivial terms of the probability we have $j_1 = 0, j_2 = 0, P = 1$, $j_1 = 1, j_2 = 0 \Rightarrow P\left[\frac{U_1}{KR_1} \leq 1\right] = P\left[R_1 \geq \frac{U_1}{K}\right]$, $j_1 = 0, j_2 = 1 \Rightarrow P\left[\frac{U_2}{KR_2} \leq 1\right] = P\left[R_2 \geq \frac{U_2}{K}\right]$, $j_1 = 1, j_2 = 1$, $P\left[\frac{U_1}{KR_1} + \frac{U_2}{KR_2} \leq 1\right] = P\left[R_1 \geq \frac{U_1}{Kx}\right] * P\left[R_2 \geq \frac{U_2}{Kx}\right] |x = 1$.

The procedure of computing the Pareto frontier is similar to the case of constant number of buses, except that now we get the end points ($U_{1,max}$ and $U_{2,max}$) from Eq.(11), and the points on the Pareto frontier must fulfill the inequality Eq.(18).

As a final note, in case there are three lines of buses, then the Pareto frontier would have the shape of a sphere sector, except for the case of $\varepsilon = 0$, in which case it would be a plane. For a higher number of bus lines, visualizing the Pareto frontier would be impossible. In that case, the optimal values of the data rates can be determined numerically (using brute force search), except for $\varepsilon = 0$ when doing so is trivial.

3.6 Static frequency sharing

Another option of resource sharing is to share statically the channels for line 1 and line 2 buses optimally, and then within the same group of users to share the available channels as before. Let's denote with K_1 the number of channels assigned to line 1 buses, and with K_2 the number of channels assigned to line 2 buses. It holds $K = K_1 + K_2$. The optimum values of K_1 and K_2 can be determined by maximizing the objective function,

$$\mathcal{F} = \log\left(U_1^{n_1} U_2^{n_2}\right) = n_1 \log U_1 + n_2 \log U_2.$$

The values of U_1 and U_2, from the above equation, according to Eq.(10) can be written as $U_1 = \frac{K_1}{\theta_1}$, and $U_2 = \frac{K_2}{\theta_2}$, where θ_1 and θ_2 are calculated in a similar way as θ_0 in Eq.(10). The optimal values of K_1 and $K_2 = K - K_1$ can be found by solving the equation

$$\frac{\partial \mathcal{F}}{\partial K_1} = 0,$$

whose solution leads to $\frac{n_1}{K_1} = \frac{n_2}{K_2}$. So, for K_1 and K_2 we have $K_1 = \frac{n_1}{n_1+n_2}K$, $K_2 = \frac{n_2}{n_1+n_2}K$, which after replacing in U_1 and U_2, we get the optimal values of the backhaul rates as

Result 5. *The maximum data rates when splitting statically the channels between line 1 and line 2 buses are given by*

$$U_{1,f} = \frac{n_1}{n_1 + n_2} \cdot \frac{K}{\theta_1},$$

and

$$U_{2,f} = \frac{n_2}{n_1 + n_2} \cdot \frac{K}{\theta_2}.$$

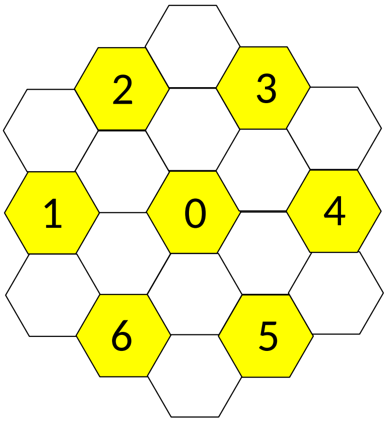

Figure 5: The system.

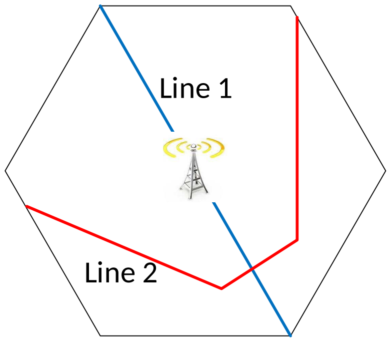

Figure 6: 2 bus lines in the cell.

4 Simulation results

4.1 Simulation setup

We consider a system with 19 hexagonal cells (each with a BTS in the center) (Fig.5), with a reuse factor of “3”. Cells shown in yellow share the same set of channels. The number of channels the BS can transmit on is $K = 100$. Our focus is cell 0 but we consider the 6 other cells to compute the inter-cell interference. The hexagon radius is 1 km. The BTS transmission power is $P_T = 40$W.

We consider an OFDM system. The physical layer parameters are based on 3GPP, and are shown in Table 1. The per-channel SINR user i receives on a given channel in during a time slot is

$$\gamma_i = \frac{\frac{P_T}{K} G_{i,0}}{N_0 + \sum_{l=1}^{6} \frac{P_T}{K} G_{i,l}},$$

where $G_{i,0}$ is the channel gain from the transmitter and cell 0. $G_{i,l}, l = 1, \ldots, 6$ are the channel gains from the BTSs of cells 1 to 6 (the interference). The channel model (gain) takes into account the path loss and shadowing (that is drawn from a lognormal distribution with parameters given in Table 1). N_0 is the additive white Gaussian noise power on the (sub)channel. More details on this model can be found in [17].

Table 1: Physical layer parameters

Noise power	-174 $\frac{dBm}{Hz}$	Shadowing average	0 dB
Shadowing s.d.	8 dB	(Sub)channel bandwidth	180 KHz
Path loss $128 + 37.6 \log 10 \left(\frac{d}{1000}\right), d > 35m$			

The system uses an adaptive modulation and coding scheme with 15 discrete

Table 2: Modulation and Coding schemes

SINR thresholds (dB)	-6.5	-4	-2.6	-1	1	3	6.6	10	11.4	11.8	13	13.8	15.6	16.8	17.6
Efficiency (bits/symbol)	0.15	0.23	0.38	0.6	0.88	1.18	1.48	1.91	2.41	2.73	3.32	3.9	4.52	5.12	5.55

Table 3: Data rate levels per channel

R (kbps)	24	36.8	60.9	96.1	141	189	237.1	306	386.1	437.4	531.9	624.8	724.2	820.3	889.2
$p_{1,k}$	0.09	0.04	0.02	0.03	0.04	0.05	0.09	0.09	0.04	0.01	0.03	0.02	0.04	0.03	0.38
$p_{2,k}$	0.06	0.23	0.11	0.1	0.1	0.08	0.11	0.07	0.02	0.005	0.015	0.01	0.02	0.01	0.06

rates [13], [14].The duration of a time slot is t = 10 ms. We consider static channel characteristics during a time slot.

Table 2 proposed in [13] and [14] gives the mapping between the SINR the mobile user receives and the corresponding efficiency (in bits per symbol) she can transmit with for the modulation and coding schemes for LTE. As we assume that the transmission scheme is OFDM, the data rate (in bps) per channel a user will experience given her SINR lies between the levels i and $i+1$ is $r = \frac{SC_{OFDM} SY_{OFDM}}{T_{sf}} e_i$, where e_i is the efficiency of level i, SC_{OFDM} is the number of data subcarriers per channel bandwidth, and T_{sf} is the subframe duration. We use the following values for these parameters [17]: $SC_{OFDM} = 12$, $SY_{OFDM} = 14$, $T_{sf} = 1$ ms.

Given the values in Table 2, the per-channel data rate of any bus rate can take one of the 15 values, as shown in the first row of Table 3.

4.2 Validation

In our model and the derivations therein (Eq.(8)) we have made some strong assumptions for tractability purposes. Namely, we have assumed that the per-channel rates for a given bus in different time slots are independent. We have also assumed that the rates seen by different rates on the same line are i.i.d. However, since the path loss is the major component of signal degradation, the bus rates in two neighboring time slots are correlated (the received signal powers will probably be relatively close). To validate our results we simulate a scenario that clearly departs from the assumptions made in the theory. We conduct simulations in MATLAB and we take the average of the metrics of interest over 1000 runs.

We perform the validation on a single line, i.e., line 1 in Fig. 6 with n buses. We assume that all the buses have a velocity of 10 m/s. We consider the case where the buses are present all the time in the cell. Basically, each bus after arriving at the end of the cell returns back and moves towards the other end of the hexagon. This would be equivalent to having the buses circulate between the two end points.

Here is a short description of our simulator. Given n, buses are initially put in arbitrary positions within the route. For every bus, at the beginning of every time slot, we calculate the per channel SINR, and based on that we determine the per-channel rate. We should remind the reader that we assume that the per-channel

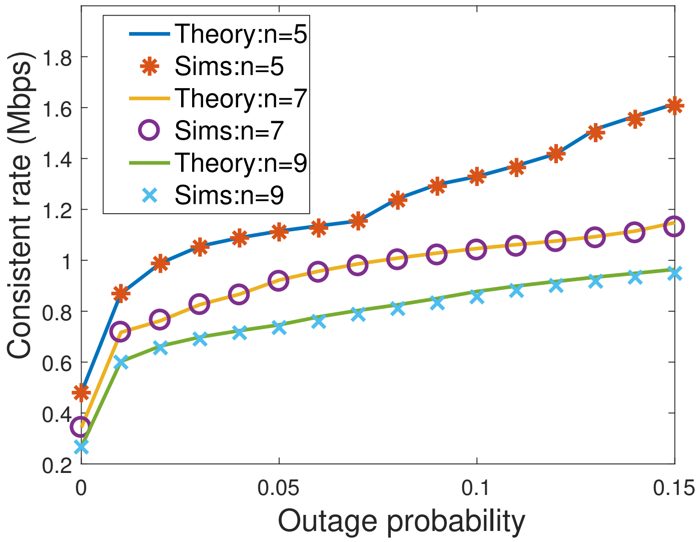

Figure 7: One bus line: Consistent rates for different constant numbers of buses vs. ε.

rate does not change within the time slot and that channels are flat. We run the simulation for $L = 100,000$ slots. After we determine the per-channel rate for all buses in all slots, we determine the probabilities of having the per-channel rate r_k, i.e., $p_{1,k}$. These values are given in the second row of Table 3 and are used for the theoretical computations.

Having run a simulation, we can easily calculate the maximum consistent rate U_{max}, for a given outage probability ε. Indeed, we know that the number of time slots in which the constraint on consistency can be violated is $\varepsilon * L$. If we order the time slots in descending order of the values of the sum over all buses of the inverses of the rate they each see, then U_{max} is determined by the time slot $\varepsilon * L + 1$, in which the corresponding realization of $U_{max}\left(\frac{1}{R} + \ldots + \frac{1}{R}\right)$ should not exceed 1.

Fig. 7 presents the theoretical and simulated maximum consistent rates vs. the outage probability for different number of buses ($n = 5, 7, 9$). As can be seen, the results are very close (the mismatch does not exceed 3-4%) despite the fact that the assumptions are not identical. This shows the usefulness of our model in predicting the performance. Furthermore, as the outage probability increases, the consistent rate increases significantly. By allowing an outage probability of only 1% we can increase the achievable backhaul rate by 50% compared to the case with $\varepsilon = 0$.

4.3 Pareto frontiers

We are now interested in determining the rate region (Pareto frontier) for the case with 2 bus lines. The routes for these buses are shown in Fig. 6. We consider scenarios with both deterministic and random number of buses. We obtain the

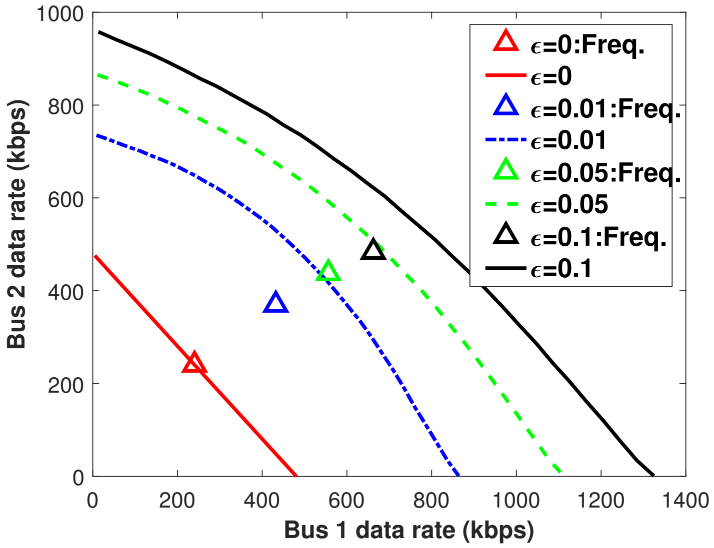

Figure 8: Pareto frontier for constant numbers of buses.

probabilities for line 2 per-channel rates the same way as we did for line 1. They are also shown in Table 3 (the third row).

We first consider $n = 10$ permanently present buses in the system, $n_1 = 5$ for line 1 and $n_2 = 5$ for line 2. We consider 4 different values of the outage probability, $\varepsilon = \{0, 0.01, 0.05, 0.1\}$. Fig. 8 illustrates the Pareto frontiers for these values of ε. We see that by increasing ε a little bit, the Pareto frontier does expand significantly. If we compare the data rates for $\varepsilon = 0$ (we must guarantee 100% of the time that rate) and $\varepsilon = 0.01$ (99% of the time the promised data rate), we can see that the rate for the later one is 50% higher. There is a diminishing return when increasing ε further.

Another interesting observation is that by splitting beforehand the channels among the two lines of buses (the corresponding results are denoted by triangles in Fig. 8), i.e., line 1 receives $K_1 = \frac{n_1 K}{n_1+n_2}$ channels at all time out of the K channels the results are worse than when we perform time sharing of resources.

We now consider the Pareto frontier for the case when both the numbers of line 1 and line 2 buses being simultaneously present in the cell are random variables. The number of line 1 buses in the system is distributed according to the following PMF: $\alpha_{1,0} = 0.2$, $\alpha_{1,1} = 0.3$, $\alpha_{1,2} = 0.1$, $\alpha_{1,3} = 0.1$, $\alpha_{1,4} = 0.15$, and $\alpha_{1,5} = 0.15$. For line 2 buses, the corresponding PMF is: $\alpha_{2,0} = 0.15$, $\alpha_{2,1} = 0.15$, $\alpha_{2,2} = 0.15$, $\alpha_{2,3} = 0.15$, $\alpha_{2,4} = 0.15$, and $\alpha_{2,5} = 0.25$. So, the maximum number of buses for each line is 5. Fig. 9 illustrates the Pareto frontiers for the same values of ε. Similar conclusions can be drawn out. The only difference is in the values of the bus backhaul rates, which are now higher than before. The reason is that more resources can be assigned to buses present at a given time instant in the cell, since

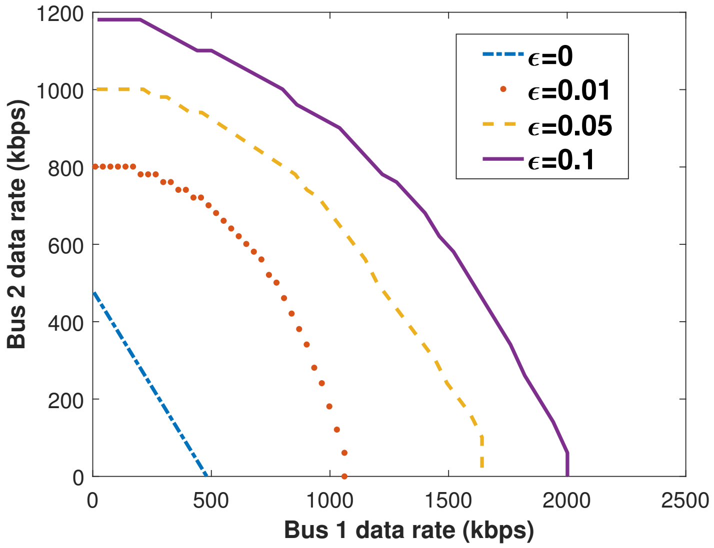

Figure 9: Pareto frontier for random numbers of buses.

some of them will not be there always.

4.4 Unused resources

Having determined the consistent backhaul rate that can be provided to buses, we proceed with characterizing the non-utilized resources. To this end, we consider the single line case where buses are moving across line 2 and consider 3 different values for ε. Fig. 10 depicts the average percentage of time a slot is not being utilized as a function of the (fixed) number of buses. Recall that the BTS strictly gives the committed backhaul rate to each bus and hence if all the buses see good channels, the proportion of time to provide them with this rate might be much lower than 1. The curves in Fig. 10 have been obtained by simulations.

Clearly, on average, the buses on the line use the resources only for a rather small portion of time (and not the whole time), and hence *the cost of consistency is very high* (almost 70% of the time in each time slot, channels are not being used for $\varepsilon = 0$, and up to 50% of the time for the other values of ε). These unused resources (time-wise) should be made available to other users of the BTS in a *best effort manner*. Interestingly, for $\varepsilon = 0$, the percentage of non-utilized channels is constant for any number of buses.

5 Conclusion

Rate consistency for bus backhauling was considered in this paper and analyzed for different scenarios using an "independence" assumption that was validated via

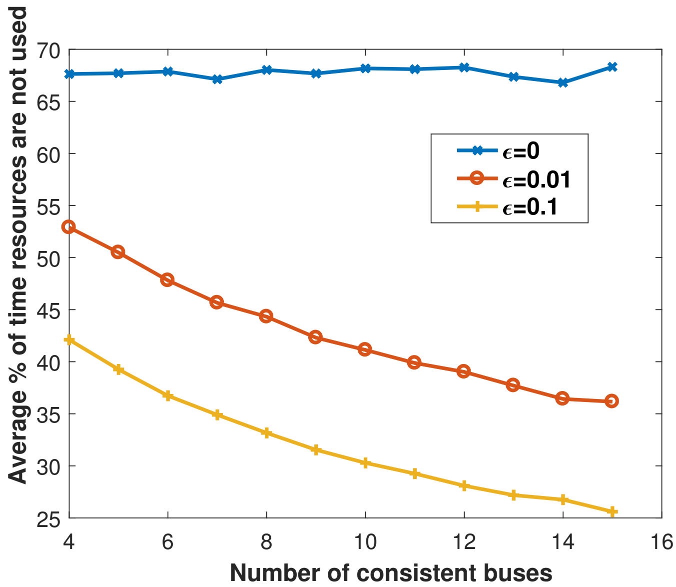

Figure 10: Non-utilized resources.

realistic scenarios. The insights are that the high price of consistency can be mitigated by allowing a small outage probability and putting a mechanism in place to offer the unused channels in a best effort manner either to the buses or to other users. As part of the future work, we plan to extend the analysis to capture any number of routes.

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