Quantifying the Benefits of Extending Electric Vehicle Charging Deadlines with Solar Generation

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***Abstract*—Significant cost reduction in recent years has made solar power an economically competitive power source in many regions today. In view of this, and the widespread introduction of electric vehicles (EVs) to the mass market, we study public EV charging stations with on-site solar generation that are backed up by conventional power from the grid. Since the carbon footprint of conventional power is higher than solar power, charging deadlines can critically affect the total solar energy available to charging stations and therefore the overall carbon footprint of the charging service. In this paper, we propose a method to quantify how much carbon footprint can be reduced as a function of the charging deadline by describing a performance-guaranteed fair power allocation algorithm in a public charging station. This enables us to study the three-way tradeoff between the charging deadline, the utility of EV owners, and the carbon footprint of EV charging. We find that our algorithm makes nearly optimal use of available green energy, while still guaranteeing that solar charging performs no worse than grid charging.**

I. Introduction

The cost of solar power generation has fallen tremendously over the past few years [1], causing global installations to increase by over 50 percent a year since 2006 [2]. In 2013, the installed cost of best-in-class rooftop solar photovoltaic systems fell to less than \$4 per watt of peak capacity for US residential customers, and studies suggest that it will fall to \$2.30 by 2015 [2]. This makes solar power an economically-viable and environmentally-friendly alternative to conventional power even if subsidies disappear entirely. In view of this steep cost reduction and the expected widespread introduction of electric vehicles (EVs) to the mass market, we envision the widespread use of solar generation to meet the EV charging demand.

Currently, only a limited number of charging stations are available for public use. However, the increasing penetration of plug-in EVs into the power grid in recent years is motivating the rapid development of charging infrastructure. We anticipate that future charging stations can be powered by on-site solar photovoltaic systems. Due to the intermittency of solar power, it must be backed up by conventional power to create a reliable supply mix. Thus, future charging stations are likely to have a grid connection in addition to on-site solar generation, which peaks at almost the same time that most EVs are parked and plugged in to chargers at public and workplace charging stations [3], [4]. We believe that solar power can become the primary supply source in these charging stations for economic and environmental reasons.

The grid connection enables the charging service provider (CSP) to give a worst-case performance guarantee to EV owners. For example, the CSP could guarantee that connected EVs are charged to a certain level by their deadlines, regardless of the incoming solar irradiation. Note that a worst-case guarantee of this type can be improved by extending the deadlines. Additionally, the charging deadlines are the key determinant of the overall carbon footprint of the charging service; the later the deadlines are, the more solar energy would be available for EV charging, reducing the use of conventional energy and the associated carbon footprint. Thus, extending the charging deadlines is also beneficial to the CSP. This motivates us to quantify the benefits of shifting the deadline by a certain value from the perspective of the CSP and EV owners.

In this paper, we study how a proposed day-ahead algorithm allocates power in a public charging station capable of simultaneously charging multiple EVs. This algorithm minimizes the carbon footprint of EV charging, provides a performance (*i.e.*, a *utility*) better than or equal to the guaranteed performance, and is *proportionally fair* to EV owners with different arrival times, deadlines, and energy demands. Assuming perfect knowledge of future EV arrivals and incoming solar irradiation, the algorithm can be used to obtain the maximum performance and the minimum carbon footprint of EV charging for specific charging deadlines. This provides a benchmark for comparing different charging deadlines in terms of the optimal performance and carbon footprint. Moreover, this algorithm could be used to obtain approximate charging schedules, given predictions of EV arrivals and departures, and incoming solar irradiation over the charging interval, which we intend to study in future work. We make three specific contributions:

- We propose a day-ahead algorithm for finding the carbon-minimizing performance-guaranteed fair power allocation to EVs in a public charging station with solar generation and multiple charging points.
- Using real traces of solar power generation, we numerically evaluate the three-way tradeoff between the carbon footprint of EV charging, the deadline, and the utility of EV owners.
- We identify three different regimes, each corresponding to a range of deadlines, and discuss whether the CSP or EV owners would benefit from extending the deadlines in each regime.

The rest of the paper is laid out as follows. In Section II we survey related work on scheduling EV charging. We present our model in Section III and formulate three optimization

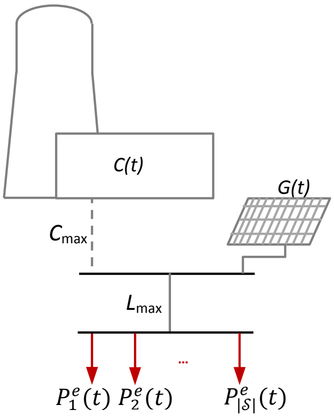

Fig. 1. Different components of the system.

problems that must be solved by the proposed algorithm to obtain the optimal carbon footprint in Section IV. We discuss the results of numerical simulations in Section V and conclude the paper in Section VI.

II. Related Work

Controlling electric vehicle charging has been extensively studied in the past. However, most existing work focuses on the potential impacts of introducing EVs into the distribution network, such as transformer overloading, branch congestion, and voltage drop. To this end, many control algorithms have been proposed ranging from real-time to day-ahead, and distributed to centralized, see for example References [5]–[9].

A substantial body of literature also studies the problem of power allocation to EV chargers in a public charging station that is supplied by the grid and collocated renewable generation [3], [4], [10], [11]. In Reference [12], the problem of charging scheduling in a charging station with stochastic EV arrivals, variable electricity prices, and intermittent renewable generation is modeled as a constrained stochastic optimization problem which can be studied using the Markov decision process framework. The objective is to minimize the mean waiting time of EVs. Reference [10] formulates the EV scheduling problem as an infinite-horizon Markov decision process with the objective of maximizing a social welfare function, which takes into account the utility of EV owners, the electricity cost associated to the charging schedule, and the penalty for failing to meet the deadlines. Reference [3] studies the same problem assuming EV arrivals, the electricity price, and available renewable energy are deterministic. The authors find the charging schedule that maximizes the operating profit of the CSP. Reference [4] proposes different online EV charging strategies to maximize a weighted average of the state of charge of plugged-in EVs. It vaguely touches on the notion of fairness in power allocation.

This work differs from these in three ways. First, our main goal is to reduce the *carbon footprint* of EV charging. Second, our approach does not guarantee that the charging deadlines will be met. Instead, we provide a worst-case guarantee based on the deadlines and the limited availability of conventional energy due to access line constraints. If deadlines cannot be met, we allocate the available energy fairly among active chargers, always providing the guaranteed worst-case performance. Third, we study the three-way tradeoff between the charging deadlines, the carbon footprint, and the utility of EV owners. This work draws on the notion of proportional fairness, originally developed for resource allocation in the Internet [13], [14], that has been recently extended to the EV charging problem [9], [15].

III. Model

Consider a public charging station with multiple identical charging points. The charging station is supplied by solar power from an on-site photovoltaic system with installed capacity of $G^{\max}$ peak Watts, in addition to conventional power from the grid as depicted in Figure 1. The access link which connects the charging station to the power grid is rated at $C^{\max}$ Watts, and power flows only in one direction, *i.e.*, from the grid to the charging station[1]. We use a time slotted model with time slots of equal length, denoted τ, and assume that demand and supply are fixed within a time slot. We denote the amount of conventional energy used in time slot t and the available solar energy from the photovoltaic system in time slot t by $C(t)$ and $G(t)$ respectively. We assume that both supply sources are connected to the load through a line rated at $L^{\max}$, where $L^{\max} \geq C^{\max}$. Hence, the charging system can provide a maximum of $C^{\max}$ without solar generation, while it can potentially provide up to $\min\{L^{\max}, C^{\max} + G^{\max}\}$ with solar generation. Note that a portion of solar generation is curtailed if $G(t) > L^{\max}$.

We assume a homogeneous EV population. We also assume that chargers are capable of charging EVs at a rate not greater than ρ Watts, and the peak charging power is fixed and independent of the state of charge (SOC) of the battery in every time slot. Let $P^e_s(t)$ be the amount of energy that the charger stores in the battery of EV s in the time slot t[2]. Suppose that the EV indexed by s arrives and connects to a designated charger in the beginning of the time slot a_s. Upon arrival, the EV driver sets the charging deadline to the end of time slot $a_s + d$, where parameter d is the same for all EVs.

We also assume that EVs remain plugged in until the deadline, and chargers become inactive and stop charging once the deadline is passed. We represent the set of time slots that the EV indexed by s was plugged in to an active charger by $T_s = [a_s, \cdots, a_s + d]$, and use $I_s(t)$ to indicate whether the EV s is connected to an active charger in the time slot t:

$$I_s(t) = \begin{cases} 1 & \text{if } t \in T_s \\ 0 & \text{otherwise} \end{cases}$$

A. Utility

We assume that user satisfaction depends on the ratio of the energy stored in the battery of an EV over the charging interval to the initial energy demand. We define the *utility* of the EV driver s as

$$u_{s,d} = \frac{\sum_{t \in T_s} P^e_s(t)}{e_s} = \frac{\sum_{t \in T} P^e_s(t) \times I_s(t)}{e_s}$$

[1]Note that $C^{\max}$ is indeed the available capacity of the most congested line or transformer in the transmission and distribution networks.

[2]To simplify the conversion between energy and power, the unit of $C(t)$, $G(t)$, and $P^e_s(t)$ is $Watt$-τ in our model, that is, the energy corresponding to 1 W of power over a duration of τ seconds.

where e_s is the initial charging demand of s in $Watt$-τ, and T is the set of time slots in the charging interval, containing all the time slots since the earliest arrival to the latest deadline. This definition takes into account the arrival time, the deadline, and the initial energy demand. Hence, $u_{s,d} = 1$ means that the EV has been fully charged by the deadline.

We adopt the notion of proportional fairness which is an axiomatically justified fairness criterion [14]. It can be shown that proportional fairness is achieved if we maximize the value of a global objective function defined as the sum of the logarithm of the utility function of the users [13]. Observe that $\log(u_{s,d})$ is infinitely differentiable, increasing, and strictly concave on its domain.

B. Carbon Footprint

We model the carbon footprint of EV charging as $f(\sum_{t\in T} C(t))$, where f is an increasing convex function of the total amount of conventional energy used over the charging interval, T. Thus, the carbon footprint of solar generation is assumed to be zero[3].

IV. Optimization Problems

In this section, we formulate a sequence of three convex optimization problems to find a carbon-minimizing performance-guaranteed proportionally fair power allocation to EV chargers. Assuming perfect knowledge of solar generation and EV arrival times, an off-line algorithm can find the lowest possible carbon footprint of EV charging by solving these optimization problems. We note that in practice these optimization problems can be solved using predictions of solar generation and EV arrival times over the charging interval, to obtain near-optimal conventional energy use and power allocation.

The solution to the first problem is the worst-case performance guarantee assuming no photovoltaic system and a deadline d. It is worst-case because a charging station without the photovoltaic system is limited to $C^{\max} \leq L^{\max}$ and has the highest carbon footprint. This first problem gives us a lower bound on the utility that each EV should expect for a given deadline d. The solution to the second problem gives us the minimal carbon footprint, *i.e.*, the minimal use of conventional power, necessary to offer at least the guaranteed performance to each user while respecting the deadline d. The solution to the third optimization problem is a proportionally fair power allocation to EV chargers that meets the guaranteed performance and also results in the optimal carbon footprint.

A. The Worst-Case Guarantee

In Optimization Problem 1, we compute the utility of EVs if conventional power is fairly distributed among the chargers given the deadline d, assuming that solar generation is zero and EV arrival times are known. The solution to this problem, denoted $u^*_{s,d}$, will be used as the worst-case guarantee that the CSP can provide to its customers. Note that the SOC of the EV cannot exceed its battery capacity because of Constraint (2).

[3]This is just a simplifying assumption. In reality, the carbon footprint of solar generation is non-zero due to emissions during manufacturing. It is straightforward to account for this by changing the objective function in Problem 2, described in Section IV; this still results in a convex problem.

Optimization Problem 1
Inputs: $d, e_s, I_s(t), C^{\max}, \mathcal{S}, \mathcal{T}$

$$\max_{\mathbf{P^e}(t)} \sum_{s\in\mathcal{S}} \log u_{s,d} \tag{1}$$

$$\text{s.t.} \quad u_{s,d} \leq 1 \qquad \forall s \in \mathcal{S} \tag{2}$$

$$\sum_{s\in\mathcal{S}} P^e_s(t) I_s(t) \leq C^{\max} \qquad \forall t \in T \tag{3}$$

$$0 \leq P^e_s(t) \leq \rho \qquad \forall t \in T,\ s \in \mathcal{S} \tag{4}$$

Observe that the grid is the only supply source in the worst case and therefore the available energy for EV charging in every time slot is limited to $C^{\max}$ ($\leq L^{\max}$).

B. Minimum Carbon Footprint for the Guaranteed Performance

In Optimization Problem 2, given the available solar energy in every time slot, we compute how much conventional energy must be drawn from the grid in every time slot so as to minimize the overall carbon footprint of EV charging and provide a utility to every EV owner that is higher than or equal to the utility guaranteed in the worst-case, *i.e.*, $u^*_{s,d}$. We denote the optimal amount of conventional energy used in time slot t by $C^*_d(t)$.

Optimization Problem 2
Inputs: $d, e_s, u^*_{s,d}, I_s(t), G(t), C^{\max}, L^{\max}, \mathcal{S}, \mathcal{T}$

$$\min_{\mathbf{P^e}(t), C(t)} f(\sum_{t\in T} C(t)) \tag{5}$$

$$\text{s.t.} \quad u^*_{s,d} \leq u_{s,d} \leq 1 \qquad \forall s \in \mathcal{S} \tag{6}$$

$$\sum_{s\in\mathcal{S}} P^e_s(t) I_s(t) \leq C(t) + G(t) \qquad \forall t \in T \tag{7}$$

$$\sum_{s\in\mathcal{S}} P^e_s(t) I_s(t) \leq L^{\max} \qquad \forall t \in T \tag{8}$$

$$0 \leq C(t) \leq C^{\max} \qquad \forall t \in T \tag{9}$$

$$0 \leq P^e_s(t) \leq \rho \qquad \forall t \in T,\ s \in \mathcal{S} \tag{10}$$

We remark that Constraint 7 is an inequality constraint since $G(t)$ represents the available solar energy which might be less than the used solar energy if solar generation is curtailed. This happens when $G(t) > \sum P^e_s(t) I_s(t)$ because it is assumed that the surplus energy cannot be transferred to the grid.

C. Fair Allocation of Available Energy to EV Chargers

In Optimization Problem 3, we find a proportionally fair power allocation to EV chargers that meets the guaranteed performance, minimizes the carbon footprint by using no more than $\sum_{t\in T} C^*_d(t)$ conventional energy over the charging interval, and respects the physical constraints of the system. Similar to the other two problems, this problem formulation depends on the arrival times of EVs, and the solar generation over the charging interval.

In the next section, we solve these optimization problems to quantify the benefits of extending the deadlines for EV owners and for the CSP.

V. Results

In this section, we solve the convex optimization problems using AMPL modelling language and Minos solver, running

Optimization Problem 3

Inputs: $d, e_s, u^*_{s,d}, I_s(t), G(t), C^*_d(t), L^{\max}, \mathcal{S}, \mathcal{T}$

$$\max_{\mathbf{P^e}(t), C(t)} \sum_{s \in \mathcal{S}} \log u_{s,d} \tag{11}$$

$$\text{s.t.} \quad u^*_{s,d} \le u_{s,d} \le 1 \qquad \forall s \in \mathcal{S} \tag{12}$$

$$\sum_{s \in \mathcal{S}} P^e_s(t) I_s(t) \le C(t) + G(t) \qquad \forall t \in T \tag{13}$$

$$\sum_{s \in \mathcal{S}} P^e_s(t) I_s(t) \le L^{\max} \qquad \forall t \in T \tag{14}$$

$$\sum_{t \in T} C(t) \le \sum_{t \in T} C^*_d(t) \tag{15}$$

$$0 \le C(t) \le C^{\max} \qquad \forall t \in T \tag{16}$$

$$0 \le P^e_s(t) \le \rho \qquad \forall t \in T,\ s \in \mathcal{S} \tag{17}$$

on a server with two six-core Intel Xeon processors, to find the minimum carbon footprint of EV charging along with the utilities of EV owners for a given population of EVs and a solar irradiation time series. For a certain size of EV population, we randomly generate ten different arrival patterns (as described below) and report the mean and the standard deviation of the parameters of interest.

Our simulation scenario involves a public EV charging station with rooftop photovoltaic arrays and a grid connection, and a finite population of customers that charge their EVs at this station. The charging station has a certain number of Level 1 EV chargers [16] with a maximum load of 1.8kW per charger. The number of chargers is assumed to be not less than the number of customers using this service; hence, a customer can always find an available charging spot upon arrival. The customers arrive at the charging station everyday after 7am, plug in their EV upon arrival, and set the deadline to x hours after their arrival, where x takes value from the set $\{4, 4.5, \cdots, 10.5, 11\}$. We set the length of every time slot to five minutes and model EV arrivals using a Poisson distribution with parameter $\lambda = 2.08$ per time slot so that on average 50 EVs arrive in two hours. We assume that all EVs have a 24kWh battery, and their SOC at the arrival time is 0.5.

We use one-minute resolution solar irradiance data from US Virgin Islands Bovoni 2 measurement station [17] and choose the panel size so as to obtain the maximum solar power of $G^{\max}$. We also set $L^{\max}$ to 90kW, and assume that the carbon footprint of conventional generating plants is proportional to the amount of energy they supply over the planning interval[4], i.e., $f(\sum_{t \in T} C(t)) = \alpha \times \sum_{t \in T} C(t)$. We study the following three cases.

A. Plenty of Solar Power

We first assume that the rooftop photovoltaic system is huge compared to the load size, permitting the algorithm to rely entirely on solar power to satisfy the guaranteed performance. Specifically, if solar energy is enough to obtain $C^*_d(t) = 0$ for every time slot t, every EV owner attains a utility higher than the guaranteed utility as a result of proportionally fair power allocation.

To see this, suppose that the installed capacity of the photovoltaic system is 80kW and the access link is rated at $C^{\max} = 10$kW; this ensures that the utility guaranteed in the

[4]We ignore the fact that carbon footprint of electricity generation might vary with time, *e.g.*, might be higher during peak periods.

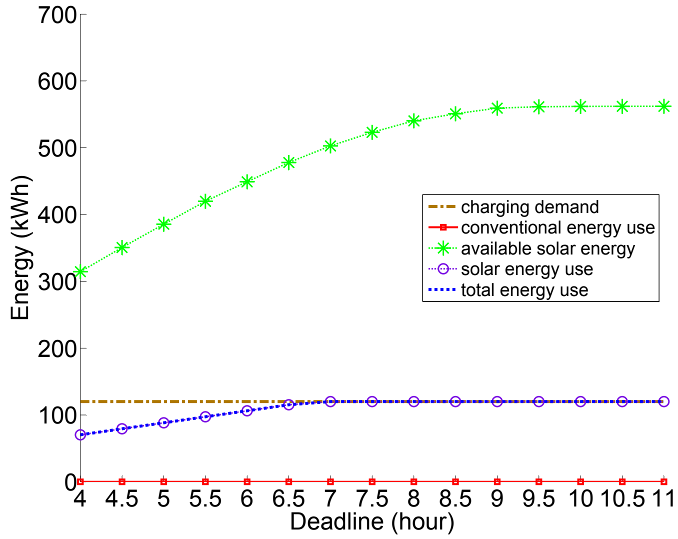

Fig. 2. Average energy supplied by different sources for different deadlines in multiple simulation runs when EV population is 10, $G^{\max} = 80kW$, and $C^{\max} = 10kW$.

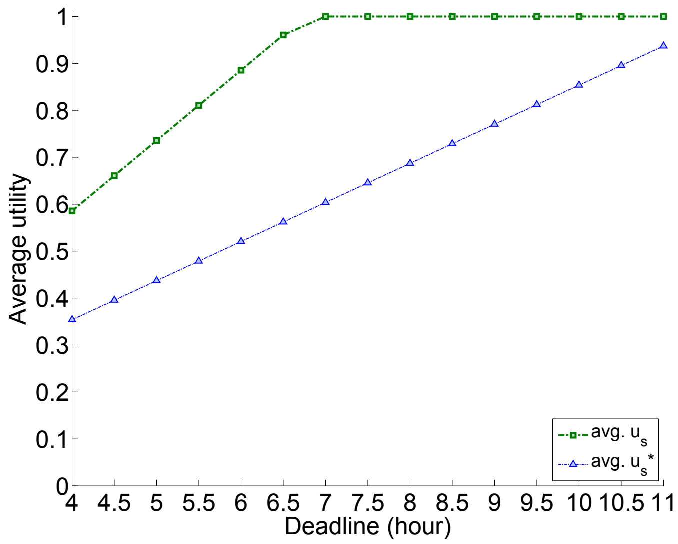

Fig. 3. The average utility, $u_{s,d}$, achieved as a result of Problem 3 and the guaranteed utility, $u^*_{s,d}$ for different deadlines when EV population is 10, $G^{\max} = 80kW$, and $C^{\max} = 10kW$.

worst-case is relatively low. Now if the charging station is comprised of 10 identical EV chargers the total EV charging demand would be $10 \times 24 \times 0.5 = 120$kWh. Figure 2 depicts the simulation results for different charging deadlines. It can be readily seen that the algorithm can meet the performance guarantee for all values of the deadline by relying entirely on solar energy; this results in zero carbon footprint and is therefore the optimal solution. Note that the use of solar energy grows with the deadline until all EVs can be fully charged; this happens when the deadline is seven hours after the arrival.

Figure 3 shows the average utility of EV owners and the worst case guaranteed utility. The proportionally fair power allocation algorithm gives every EV owner a utility which is much higher than the guaranteed one. Note that the average utility is one when the deadline is seven hours after the arrival, confirming that EVs all fully charged at this point.

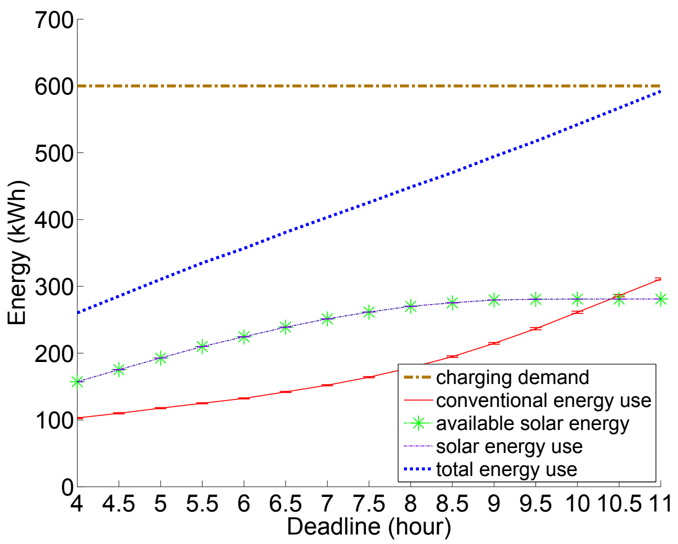

Fig. 4. Average energy supplied by different sources for different deadlines in multiple simulation runs when EV population is 50, $G^{\max} = 40kW$, and $C^{\max} = 50kW$. Note that error bars represent one standard error.

Since the carbon footprint of EV charging is zero for all deadlines, EV owners can attain a higher utility be extending their deadlines from four hours to seven hours without increasing the carbon footprint. A further extension of the deadlines does not benefit anyone in this case.

B. Limited Solar Power

We now turn our attention to the case that the solar energy is not sufficient; thus, conventional energy must be used in addition to solar energy to satisfy the worst-case performance guarantee. This would be the case, for example, when the installed capacity of the photovoltaic system is 40kW, the access link rated at 50kW, and the EV population is 50 (the total charging demand is $50 \times 24 \times 0.5 = 600$kWh).

Figure 4 shows the simulation results for different charging deadlines. Observe that the amount of solar energy available for EV charging increases with the deadline until sunset, and all the available solar energy is used for EV charging by the algorithm. However, since the available solar energy is not sufficient to satisfy the guaranteed performance, the use of conventional energy also increases with the deadline. In fact, its rate of increase is higher than that of solar energy when the deadline is shifted to the evening.

The blue curves with triangle and cross markers in Figure 5 show the average utility that EV owners attain as a result of Problem 3 and the utility guaranteed to them in Problem 1 respectively. Observe that the average utility of EV owners is always higher than the guaranteed utility and it increases with the deadline, but never reaches one.

In this case, extending the charging deadline increases the utility of EV owners but this comes at the price of increasing the carbon footprint especially when the deadline moves to the evening. We believe that this tradeoff is very useful for policy making. For example, if the carbon footprint of the charging service should not exceed a certain threshold, then the CSP would be required to put a limit on the charging deadlines.

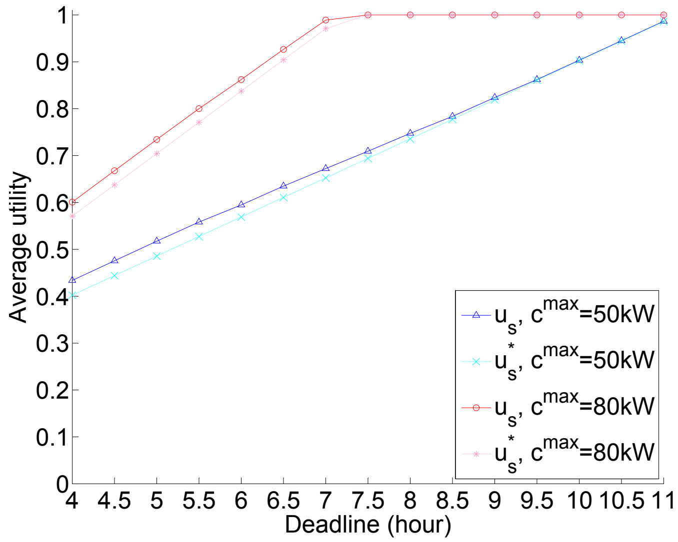

Fig. 5. The average utility and the guaranteed utility for different deadlines and two values of $C^{\max}$ when EV population is 50 and $G^{\max} = 40kW$.

C. Plenty of Conventional Power

We finally study the case when the solar energy is insufficient (similar to the previous case) but this time the capacity of the access link is such that a considerably higher utility can be guaranteed to EV owners in the worst case. To see this, we need to increase the capacity of the access link to 80kW, while the rest of the parameters are identical to the previous case described in Section V-B.

To differentiate this case from the previous case, we first look at the utilities. The red curves with circle and asterisk markers in Figure 5 show the average utility that EV owners attain as a result of Problem 3 and the utility guaranteed to them in Problem 1 respectively. Observe that EV owners attain a higher utility as we increase the deadline, until the deadline reaches 7.5 hours after the arrival. After this point all EVs are fully charged and the average utility is one.

The Effect of Charging Deadlines: Figure 6 shows the simulation results for different charging deadlines. We witness three different behaviours in this case identified by three regimes. The first regime corresponds to the case that EVs are not fully charged and the optimal power allocation uses almost all the available solar energy in addition to conventional energy to satisfy the guaranteed performance. Thus, the same tradeoff between the average utility of EV owners and the carbon footprint of EV charging exists in this regime. In the second regime all EVs are fully charged and increasing the deadline only results in more solar energy that can be used for charging EVs. Therefore, the algorithm replaces some amount of conventional energy with solar energy to further reduce the carbon footprint without affecting the utility of EV owners. In this regime, extending the deadlines does not benefit the EV owners but it helps reduce the overall carbon footprint of EV charging. Finally, in the third regime extending the deadlines does not benefit the EV owners and does not reduce the carbon footprint of EV charging since there is no more solar energy available late in the evening.

Figure 7 clearly shows the difference between these three regimes. In particular, it shows the ratio of the used solar

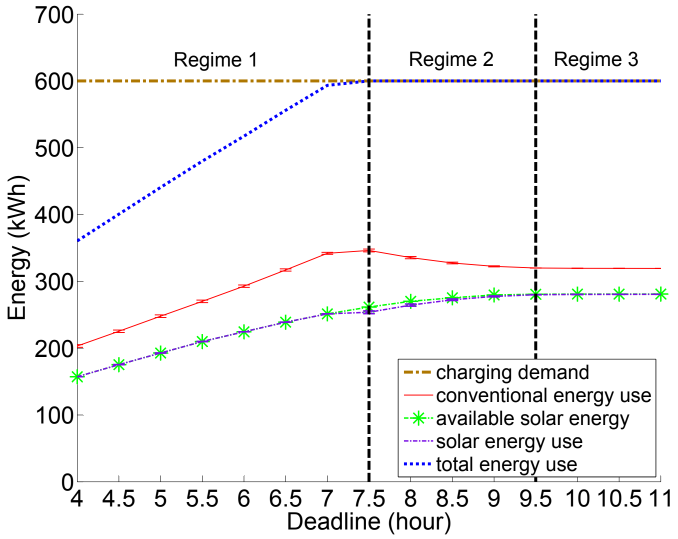

Fig. 6. Average energy supplied by different sources for different deadlines in multiple simulation runs when EV population is 50, $G^{\max} = 40kW$, and $C^{\max} = 80kW$. Note that error bars represent one standard error.

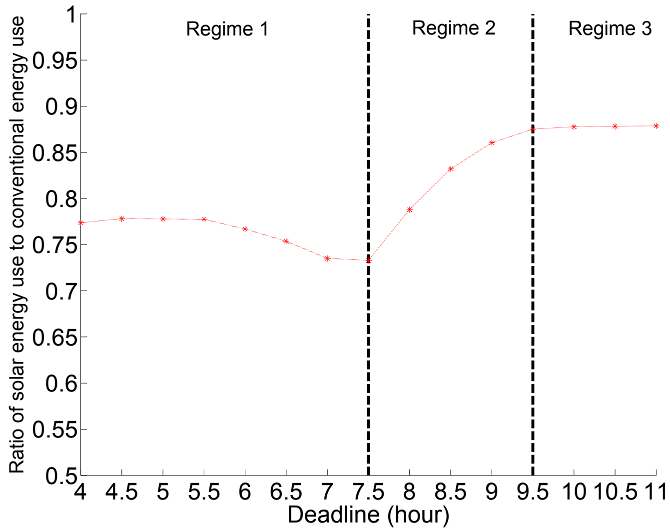

Fig. 7. Regime switching happens as the deadline increases. In this case the EV population is 50, $G^{\max} = 40kW$, and $C^{\max} = 80kW$.

energy to the used conventional energy by the algorithm. It can be readily seen that this ratio does not vary much in the first regime because the usage of both energy sources increases at approximately the same rate. However, in the second regime this ratio grows noticeably as the algorithm replaces conventional energy by solar energy. Finally in the last regime this ratio converges to an asymptotic value.

VI. Conclusion

Solar power is becoming competitive with conventional power in many jurisdictions. Powering public EV charging stations at workplaces using local solar generation is a promising application because solar power peaks at almost the same time that utilization of these stations peaks, creating an opportunity to absorb the available solar energy without the need for storage and reduce the carbon footprint of EV charging. In this context, we quantify costs and benefits of shifting EV charging deadlines. We propose an off-line carbon-minimizing proportionally fair power allocation algorithm with a guaranteed worst-case performance for a public charging station with multiple charging points. We run numerical simulations to show how extending the deadlines could change the utility of EV owners and the carbon footprint of EV charging if this algorithm is adopted. In future we plan to extend this work to design an on-line scheduling algorithm that uses predictions to obtain a near-optimal result. We also plan to include inelastic loads in our model.

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