Joint Configuration of Routing and Medium Access Parameters in Wireless Networks*

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***Abstract*— In this paper, we study the joint configuration of routing and medium access control (MAC) parameters in fixed wireless networks. Due to the complexity of the problem, we consider a simple slotted ALOHA MAC protocol for link layer operation. We model the link rate of the slotted ALOHA system under a saturation assumption and use a signal to interference plus noise ratio (SINR) based interference model via the concept of *conflict set*. We formulate a joint routing and MAC (JRM) optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of routing and MAC parameters. The JRM optimization problem is a non-convex optimization problem and we solve it by an iterated optimal search technique. We validate our approach via simulation and illustrate the potentially high throughput gains that can be obtained by using our joint configuration technique.**

I. Introduction

Although the worldwide success of the Internet is partly due to the simplicity and robustness of its layered network architecture, this architecture is not flexible enough for multi-hop wireless networks. Cross-layer approaches have been proposed [1], [2] to enhance the adaptability and performance of these networks. Cross-layer design provides an opportunity to optimize performance by jointly tuning parameters at the different layers.

One of the critical performance metrics in multi-hop wireless networks is goodput. In a multi-hop wireless network, goodput is highly dependent on the configuration of routing and medium access parameters and on their interactions, see for example [1] in the case of a (conflict-free) scheduled network. Configuring a wireless network based on random access is much more difficult, and one might be tempted to simply use a so-called *default* configuration comprised, for example in the case of slotted ALOHA, of a minimum hop routing and equal attempt probability at all nodes. While one would expect that joint configuration of routing and access parameters of a random access network can provide better performance than the default configuration, there is no clear indication so far on how much improvement can be achieved by joint design and how to configure the parameters jointly.

In this paper, we study the joint configuration of routing and MAC parameters in multi-hop slotted ALOHA wireless networks to maximize the minimum throughput of the flows. We want to provide some insights about the performance gain obtained by joint design, and on how to configure routing and MAC parameters. We consider a slotted ALOHA system under an interference model based on SINR, where nodes are stationary and traffic flows are static. Our contributions are as follows:

- We model the effective link rate under a so-called saturation assumption and use an interference model based on SINR via the concept of *conflict set*. This link rate model is found to be very complex and is not a convex function of its parameters.
- We formulate an optimization problem to determine the optimal max-min throughput of the flows and the optimal configuration of the routing and MAC parameters. Since the link rate model is not convex, neither is the optimization problem.
- We solve the optimization problem numerically by using the iterated optimal search (IOS) technique.
- We validate the configurations obtained via our model by simulation. We show that if we use the routing and access parameters calculated by the model in a real network, we can reach the maximum throughput calculated by the model and that any larger value will make the network unstable.
- We provide numerical and simulation results for various scenarios. These results show that performance gains as large as 67% can be obtained by configuring the network using our model instead of using the default configuration.

The rest of the paper is organized as follows. Some related work is reviewed in Section II. Section III presents the network and link model in detail. We formulate the JRM optimization problem in Section IV and present the solution technique in Section V. The model is validated in Section VI and numerical and simulation results are presented in Section VII. Section VIII concludes this work.

II. Related Work

Since the early 1990's, researchers have tried to address the problem of joint routing and MAC for multi-hop ALOHA wireless networks [3], [4]. In [3], a nonlinear joint optimization

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problem is formulated using a simple interference model and solved by decoupling the routing and the MAC problems. For the routing problem, a heuristic is used to find the minimum hop path with low interference and then the MAC problem is solved by an iterative numerical method. In [4], the problem is solved by forcing the attempt probabilities to be fixed and equal for all nodes. This transforms the original problem into a linear program which can be easily solved. In both papers, the authors have decoupled the MAC and routing problems to get some workable solution. In this paper, we consider a problem based on a more sophisticated interference model and a slightly different objective function, and solve the joint problem.

In [5], experiments on an IEEE 802.11b based multi-hop wireless testbed are performed to investigate the performance of minimum hop routing. It is shown that minimum hop routing does not perform well. This is also the conclusion of a study [6] for scheduled multi-hop networks.

Loosely coupled cross-layer design between the network layer and MAC layer is addressed in many papers by designing different routing metrics [7], [8], [9]. The purpose of these loosely coupled cross-layer design is to determine the optimal route of a newly arriving session or an existing session after a route failure by computing the metric value of different paths based on MAC layer information. Cross-layer design based on routing metrics improves throughput performance by exploiting the MAC layer information and is easy to implement distributively. However, it cannot achieve the optimal throughput performance since routes of the existing sessions and MAC parameters are not adapted to the routing impact of a new arriving or failure session. Different from the loosely coupled cross-layer design based on a routing metric based, we focus on a tightly coupled joint routing and MAC design to achieve the optimal throughput performance.

Tightly coupled joint design of routing and scheduling (as opposed to random access MAC) is addressed in many papers (For example see [1], [10], [11], [12]).

III. System Model

A. Network Topology

Consider a wireless network consisting of N stationary nodes with known locations. The set of nodes is denoted by $\mathcal{N}$. Each node has an omni-directional antenna and cannot transmit and receive at the same time. The transmission power of all the nodes is fixed and assumed to be the same for all nodes. Let $\mathcal{L}$ be the set of directed links in the network and $L = |\mathcal{L}|$. A directed link $l \in \mathcal{L}$ is also represented as (l_o, l_d), where l_o and l_d are the originating and destination nodes of the link. We denote the sets of links coming into and going out of node n by $\mathcal{L}_n^I$ and $\mathcal{L}_n^O$. A summary of important symbols is given in Table I for easy reference.

TABLE I

A summary of important symbols

Symbol	Definition
$\mathcal{N}$	Set of nodes
$\mathcal{L}$	Set of directed links
$\mathcal{L}_n^O$	Set of links going out of node n
$\mathcal{L}_n^I$	Set of links coming into of node n
$\mathcal{F}$	Set of data flows
f_s	Source node of flow f
f_s	Destination node of flow f
λ_f	Source rate of flow f
$\boldsymbol{\lambda}$	Source rate vector of the flows
π_n	Attempt probability of node n in a given time slot
$\boldsymbol{\pi}$	Attempt probability vector of the nodes
q_{nf}	Selection probability of flow f by node n
$\mathbf{q}$	Flow selection probability vector
r_{nfl}	Selection probability of link l for flow f by node n
$\mathbf{r}$	Link selection probability vector
τ_{fl}	Transmission probability of flow f on link l
$\boldsymbol{\tau}$	Transmission probability vector
c_{fl}	Effective rate of flow f in link l
y_{fl}	Traffic rate of flow f in link l
$\mathbf{y}$	Traffic rate vector of the flows in the links
z	Max-min throughput
Δ_1	Default configuration 1
Δ_2	Default configuration 2

B. Wireless Channel and Interference Model

The channel gain of a link is assumed to be quasi time invariant. The normalized[1] channel gain between nodes n_1 and n_2, $G_{n_1n_2}$, is given by $(d_{n_1n_2}/d_0)^{-\eta}$, where $d_{n_1n_2}$ is the distance between nodes n_1 and n_2, d_0 is a reference distance in the far field of the transmit antenna, and η is the path loss exponent. We assume that all the nodes use the same modulation and coding scheme characterized by a unit rate and an SINR threshold γ. A directed link between n_1 and n_2 exists if the signal-to-noise ratio (SNR) for the link, assuming no nodes other than n_1 are transmitting, is greater than γ, i.e.,

$$\frac{G_{n_1n_2}P_t}{N_0} \geq \gamma \tag{1}$$

where N_0 is the received background noise power and P_t is the transmission power. Time is slotted and the size of a packet is fixed and corresponds to the duration of one time slot. A packet sent by n_1 in a given time slot is considered to be successfully received by the receiver n_2 if the received SINR is higher than γ. Thus, a packet transmission from node n_1 to n_2 is successful if

$$\frac{G_{n_1n_2}P_t}{N_0 + \sum_{n' \neq n_1} G_{n'n_2}P_tY_{n'}} \geq \gamma \tag{2}$$

where by convention $G_{nn} = \infty$, $Y_{n'}$ is a binary variable taking on value 1 if node n' transmits in the given slot and 0 otherwise.

C. Routing, Medium Access, and Retransmission Strategy

All the nodes access a single channel according to a slotted ALOHA MAC protocol. There are F data flows in

[1]We assume that the channel gains are normalized to simplify our notations but this assumption is not necessary to carry out our study.

the network, denoted by set $\mathcal{F}$. A data flow f is characterized by its source f_s and its destination f_d. The traffic rate at the source of flow f is constant and denoted by λ_f. The collection of λ_f variables is represented by the source rate vector $\boldsymbol{\lambda}$.

Each node maintains a separate infinite queue for each flow. Clearly the set of flows that a node has to relay is a function of the routing and is only a subset of $\mathcal{F}$. We make a strong assumption that there are always packets of each flow available at each node so that a node that wants to transmit can always do it. This is what we call the *saturation assumption* in the following. This assumption might seem unrealistic since if not handled properly, it could mean that a node can generate packets for a flow even though this flow is not routed through the node. We will add constraints to the optimization problem in Section IV-C to guarantee that this cannot happen.

The operation of the network is described by the following random variables. First let π_n denote the probability that node n will try to access the channel in a given slot, i.e., the attempt probability, and the corresponding probability vector $\boldsymbol{\pi}$. Given that node n does try to access the channel, we then denote the conditional probability that it will select flow f by q_{nf} with the condition $\sum_{f\in\mathcal{F}} q_{nf} = 1$. The collection of q_{nf} variables is represented by the flow selection probability vector $\mathbf{q}$. Finally, given that the node attempts to access the channel and that it selects flow f, we let r_{nfl} denote the conditional probability that it will send the packet on link l again with $\sum_{l\in\mathcal{L}_n^O} r_{nfl} = 1$. The vector corresponding to these variables is denoted by $\mathbf{r}$. Hence, in our model, the routing of the flows is defined by $\mathbf{q}$ and $\mathbf{r}$, and the channel access rate by $\boldsymbol{\pi}$.

At each slot, node n first generates a Bernoulli variable with probability π_n. If the result is 1, it generates a variable from a non uniform discrete distribution with probability q_{nf} to choose flow f to transmit, and then another variable with probability r_{nfl} to select link l to transmit the packet. We assume that a transmitter knows immediately at the end of the current slot whether its transmission is successful or not. We consider a delayed first transmission (DFT) retransmission policy, where the transmitting node keeps a copy of the packet in the queue that it is transmitting. This copy is deleted if the transmission is successful; otherwise it is retransmitted when the transmitter selects that flow again.

IV. Problem Formulation

A. Effective Link Rate

If they were alone, two nodes could communicate at some nominal rate C determined by the physical layer parameters. The presence of other nodes and the MAC policy will reduce the nominal rate to a lower value because of collisions and retransmissions. This is referred to as the *effective* link rate.

Let τ_{fl} be the probability that a packet of flow f will be transmitted on link l in a given time slot. It is given by

$$\tau_{fl} = \pi_n q_{nf} r_{nfl} \qquad \forall l \in \mathcal{L}_n^O. \tag{3}$$

The collection of τ_{fl} is called the transmission probability vector, denoted by $\boldsymbol{\tau}$. The probability that node n will transmit a packet of flow f on any of its outgoing links is given by

$$\pi_n q_{nf} = \sum_{l\in\mathcal{L}_n^O} \tau_{fl} \tag{4}$$

and the probability that node n will transmit a packet of any flow on any of its outgoing links is given by

$$\pi_n = \sum_{l\in\mathcal{L}_n^O, f\in\mathcal{F}} \tau_{fl}. \tag{5}$$

Because nodes are able to know immediately whether a collision has occurred, the effective rate c_{fl} of flow f in link l can be expressed as

$$c_{fl} = C\tau_{fl} p_l^s \tag{6}$$

where p_l^s is the probability that a packet can be transmitted successfully on link l, i.e., that the SINR at l_d will be greater than the threshold γ.

B. Computation of p_l^s

The main difficulty of the model is the calculation of p_l^s. In what follows, we drop the link index l in order to simplify the notation and carry out the discussion for a given link. First, we define a *conflict* set σ for the link as a set of nodes such that the transmission on the link will fail if *all* the nodes $j \in \sigma$ are transmitting during the slot. In that case, we say that the conflict set is *active* during the slot. Because each node decides whether or not it will transmit independently of all the other nodes, the probability $P\{\sigma\}$ that a conflict set is active is given by

$$P\{\sigma\} = \prod_{j\in\sigma} \pi_j. \tag{7}$$

We number the conflict sets with the superscript k to represent the k^{th} conflict set of the link. Let ν be the number of conflict sets for the link. We also represent the event $\{\text{Conflict set } \sigma^k \text{ is active}\}$ by σ^k. The probability $1-p^s$ that the transmission will fail (on the given link) because of the SINR constraint is not satisfied is given by

$$1 - p^s = P\left\{\bigcup_{k=1}^{\nu} \sigma^k\right\}. \tag{8}$$

In other word, the transmission fails if *any one* of the conflict sets is active during the slot. In general, as the conflict sets of the link are *not* independent, we have

$$1 - p^s = \sum_{j=1}^{\nu} (-1)^{j+1} \sum_{k_1<\ldots<k_j} P\left\{\sigma^{k_1} \cap \ldots \cap \sigma^{k_j}\right\} \tag{9}$$

where $P\left\{\sigma^1 \cap \sigma^2 \cap \ldots \cap \sigma^j\right\}$ is the probability that all the sets σ^1, $\ldots$ σ^j are active and is given by

$$P\left\{\sigma^1 \cap \sigma^2 \cap \ldots \cap \sigma^j\right\} = \prod_{i\in\sigma^1\cup\sigma^2\cup\ldots\cup\sigma^j} \pi_i \tag{10}$$

again from the independent decisions of each node to transmit or not. The calculation of the effective rate for a given link l is then made up of two parts. The first one is the enumeration of all the conflict sets of the link. This depends on the parameters of the physical layer and on the position of the nodes, but does not depend on the $\boldsymbol{\pi}$ or $\boldsymbol{\tau}$ variables. The second step is the evaluation of the polynomial in $\boldsymbol{\pi}$ given by (9). This calculation has to be done whenever the values of the $\boldsymbol{\pi}$'s change, for instance during an iterative optimization procedure. The effective rate c_{fl} is then a polynomial function of the $\boldsymbol{\tau}$ and $\boldsymbol{\pi}$ variables and is denoted by $c_{fl}(\boldsymbol{\tau}, \boldsymbol{\pi})$ in the next sections.

As discussed, the computation of p_l^s is based on all the conflict sets for link l. However, the computation can be done faster by considering only minimal conflict sets, i.e., conflict sets that are no longer conflict sets if any node is removed from them.

C. Problem Statement

We can now state the cross-layer design problem. Denote the traffic rate of flow $f \in \mathcal{F}$ on link $l \in \mathcal{L}$ by y_{fl} and the corresponding traffic rate vector by $\mathbf{y}$. Recall that the source and destination nodes of flow $f \in \mathcal{F}$ are f_s and f_d. Let z be the minimum throughput of all the flows. We normalize the physical transmission rate to $C = 1$. The JRM optimization problem to maximize the minimum throughput of the flows is given by

$$\max_{\boldsymbol{\tau}, \boldsymbol{\pi}, \boldsymbol{\lambda}, \mathbf{y}} \quad z \tag{11}$$

$$z \leq \lambda_f \quad \forall f \in \mathcal{F} \tag{12}$$

$$\sum_{l \in \mathcal{L}_n^O} y_{fl} - \sum_{l \in \mathcal{L}_n^I} y_{fl} = \begin{cases} \lambda_f & \text{if } n = f_s \\ -\lambda_f & \text{if } n = f_d \\ 0 & \text{otherwise} \end{cases}$$

$$\forall n \in \mathcal{N}, f \in \mathcal{F} \tag{13}$$

$$y_{fl} \leq c_{fl}(\boldsymbol{\tau}, \boldsymbol{\pi}) \quad \forall f \in \mathcal{F}, l \in \mathcal{L} \tag{14}$$

$$\sum_{l \in \mathcal{L}_n^O, f \in \mathcal{F}} \tau_{fl} = \pi_n \quad \forall n \in \mathcal{N} \tag{15}$$

$$0 \leq z, \boldsymbol{\lambda}, \mathbf{y} \tag{16}$$

$$0 \leq \boldsymbol{\tau}, \boldsymbol{\pi} \leq 1. \tag{17}$$

The objective function in (11) and constraints in (12) ensure that the minimum throughput of the flows is maximized. The flow conservation constraints in (13) guarantee that the outgoing and incoming traffic of a flow are equal at each intermediate node, that the outgoing traffic of a flow is equal to the source rate at the source node, and that the incoming traffic of a flow is equal to the source rate at the destination node. This ensures that intermediate nodes cannot create flows, which is not forbidden by the saturation assumption of section III-C. The link rate constraints in (14) ensure that the traffic rate on a link is not larger than the link rate for each flow. The equality constraints in (15) relate the attempt probabilities to the transmission probabilities. They could be used to remove the $\boldsymbol{\pi}$ variables from the problem but they are left in for the sake of clarity. Equations (16) and (17) are the bounds on the variables.

The problem in (11-17) is a nonlinear optimization problem because the constraints in (14) have a strong nonlinear dependence on the $\boldsymbol{\pi}$ variables. Furthermore, constraints in (14) are not convex since both sides of the constraints turn out as posynomials when $c_{fl}(\boldsymbol{\tau}, \boldsymbol{\pi})$ is expressed in variables $\boldsymbol{\tau}$ and $\boldsymbol{\pi}$ using (6), (9) and (10) [14]. Thus, finding a global optimal solution is a challenge. The solution determines the optimal $\boldsymbol{\pi}^*$, $\boldsymbol{\tau}^*$ and z^*. The optimal values for $\mathbf{q}^*$ and $\mathbf{r}^*$ can be determined from $\boldsymbol{\pi}^*$ and $\boldsymbol{\tau}^*$ using (3) and (4). It is not difficult to show that the solution of this nonlinear set of equations is unique and that it satisfies the normalizing conditions of section III-C.

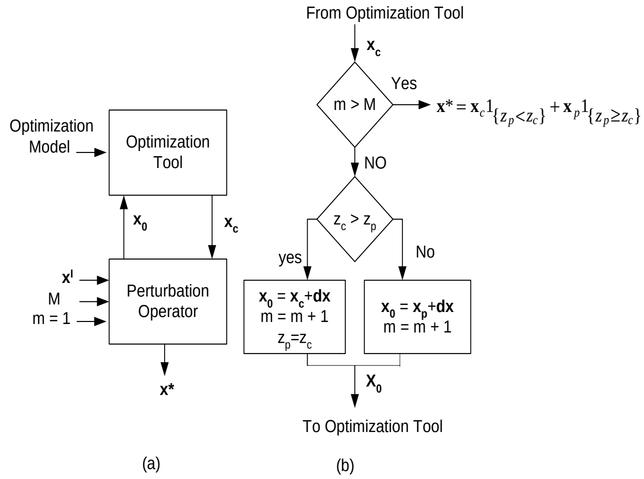


Fig. 1. Iterated optimal search: (a) relation between local search and perturbation (b) perturbation algorithm

V. Solution Technique

Since the JRM problem is nonlinear and not convex, computing a global optimum is difficult if not impossible for large networks. At this point, we chose to solve it by the IOS technique which is an iterated local search technique [13] and focus on small to medium size networks.

For a given problem characterized by its input variables, this technique finds a sequence of local maxima by starting from different initial values at each iteration. The main feature of the method is that the initial values of a local search are chosen using the best solution of the previous iterations. To describe the operation, denote the vector of variables of the optimization problem by $\mathbf{x}$. At each iteration, the optimization tool finds a local optimum taking a new vector $\mathbf{x}_0$ as the initial values. At the end of iteration m, the perturbation operator computes the new initial values of the variables for iteration $m+1$ using $\mathbf{x}_c$, the optimal solution of the m^{th} iteration. For the first iteration, the perturbation operator sets $\mathbf{x}_0 = \mathbf{x}^I$, where $\mathbf{x}^I$ is some initial set of variables. We perform M iterations for each initial vector $\mathbf{x}^I$ and try 3 different initial $\mathbf{x}^I$ vectors per problem. More precisely at the beginning of the $(m+1)^{th}$ iteration, $\mathbf{x}_0 = \mathbf{x}_c 1_{\{z_c > z_p\}} + \mathbf{x}_p 1_{\{z_c \leq z_p\}} + d\mathbf{x}$, where $\mathbf{x}_c$ is the solution of the m^{th} iteration that provides the value of the objective function z_c, $\mathbf{x}_p$ is the best solution among the first $m-1$ iterations that provides the value of the objective function z_p, and $d\mathbf{x}$ is a perturbation vector given by

TABLE II

INITIALIZATION OF THE DIFFERENT VARIABLES

	z	$\boldsymbol{\lambda}$	$\mathbf{y}$	$\boldsymbol{\pi}$	$\boldsymbol{\tau}$
$\mathbf{x}_1^I$	0	0.02	0.01	0.1	0.025
$\mathbf{x}_2^I$	0	0.05	0.025	0.15	0.02
$\mathbf{x}_3^I$	0	0.1	0.05	0.2	0.05

TABLE III

PHYSICAL LAYER PARAMETERS

Parameter	Network 1	Network 2
Transmission power (dBm)	0	0
SINR threshold (dB)	15	6.4
Noise power (dBm)	-100	-100
Path-loss exponent	4	3
Far-field crossover distance (m)	0.1	0.1

$d\mathbf{x} = \alpha\mathbf{x}^I$, where α is a uniformly distributed random variable in the interval $[-a, a]$. We try three values of a, and hence three perturbation vectors, for each initial vector $\mathbf{x}^I$. Hence, for each problem, we obtain nine values of the objective function and we select the largest one as the solution to our problem. Fig.1 illustrates this technique.

We use MINOS 5.51 [15] to compute the local maxima at each iteration of the IOS algorithm. In our study, we let $M = 30$, the 3 values of a to be 0.25, 0.5, and 1. The three initial $\mathbf{x}^I$ vectors are given in Table II.

VI. MODEL VALIDATION

The link rate model that we use to compute the optimal max-min throughput, the routing and the MAC configuration are based on the assumption that the queues are saturated. This is not always the case in practice so that it is important to validate this saturation assumption. This can be done by simulating a network configured with the optimal parameters calculated by the algorithm. We consider that the model is validated if the computed throughput can be achieved by the simulation but cannot be further increased. For the simulation, $\mathbf{q}^*$ and $\mathbf{r}^*$ are calculated from (3) and (4) from the optimal configuration $\boldsymbol{\tau}^*$ and $\boldsymbol{\pi}^*$.

A. Network and Algorithm Parameters

We use two random networks with different flow sets, yielding 10 different scenarios. Because the computation time of the link rates from (6) increases very fast with the number of nodes, both networks are limited to 10 nodes. Two sets of different physical layer parameters are given in Table III. The two networks are shown in Fig. 2 (a) and Fig. 3 (a) with only the odd numbered directed links for clarity. The directed links in the opposite direction have the following even numbers. The links are determined using (1). A scenario is characterized by the network (either network 1 or 2) and a flow set. The different scenarios are shown in Table IV.

B. Simulator Setup

The average rates of the sources are all set to equal values and their traffic is assumed to be Poisson. The node decision to transmit or not and the selection of which flow to transmit

TABLE IV

THE SCENARIOS

Set	$\lvert\mathcal{F}\rvert$	Network 1	Network 2
1	2	$\{(6,4),(8,9)\}$	$\{(1,5),(7,6)\}$
2	3	$\{(3,4),(8,5),(6,10)\}$	$\{(7,5),(9,6),(6,5)\}$
3	4	$\{(4,6),(8,9),(7,4),(9,2)\}$	$\{(4,1),(1,5),(5,6),(6,9)\}$
4	5	$\{(5,2),(6,4),(9,8),(10,7),(3,9)\}$	$\{(9,5),(1,6),(6,5),(5,1),(7,6)\}$
5	9	$\{(i,9)\} : i = 1 \ldots 10, i \neq 9$	$\{(i,6)\} : i = 1 \ldots 10, i \neq 6$

are implemented in the simulation as described in the system model. When the source rate is low, a node may not always have a packet of the selected flow to transmit. In that case, the node does not transmit.

Each node maintains a separate queue for each flow with a buffer of size 1000 packets. In the simulator, the number of packets in a queue is increased by one if a new packet arrives, decreased by one if a transmission is successful and kept unchanged if a transmission is unsuccessful. Since a separate queue is maintained for each flow, this strategy is equivalent to the DFT retransmission strategy mentioned earlier. The simulation is done using C++.

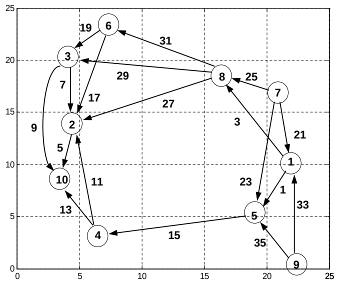
C. Determining the Max-min Throughput of a Network Configuration

For a particular source rate, the packet loss probability of each queue is estimated from the ratio of the number of loss packets and the number of packets that arrived at the queue over a window of 4×10^7 slots after a network loading time of 10^6 slots. The total simulation time is then 4.1×10^7 slots. To determine the max-min throughput with a small error, the source rate is increased from a starting value λ_0 by small increments of 0.0001 till the system becomes unstable. The system stability is checked at each step using the statistical test described in Appendix.

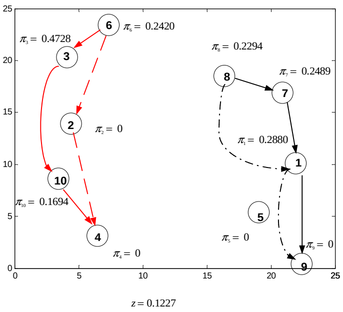
The comparison between our numerical results and the simulation results is summarized in Table V. The column labeled "Numerical" contains the maximum throughput computed by the JRM algorithm. The column labeled "Simulation" contains the maximum and minimum values of the largest stable throughput obtained over 10 simulation runs. The difference between the numerical and simulation results is less than 1% in most of the cases and the maximum difference is found to be 4.25%. Based on this, we can consider that the model has been validated.

VII. THE ADVANTAGES OF JOINT CONFIGURATION

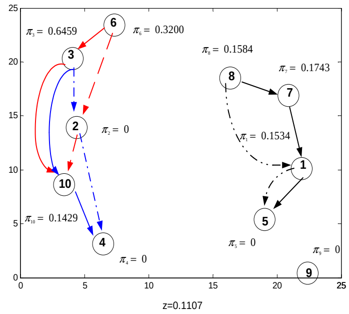
We are now in a position to quantify the performance gain that can be obtained by a joint routing and MAC configuration of the network over what could be obtained using a default configuration. In all cases, the default configurations use minimum hop routing and the same attempt probability at all nodes. If the number of minimum hop paths is more than one, the shortest distance path is chosen. If more than one flow



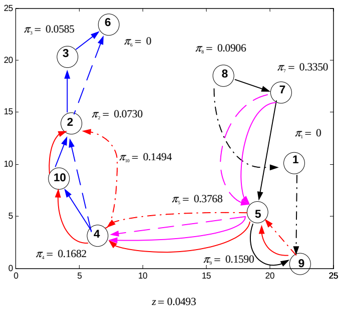
(a) Network 1



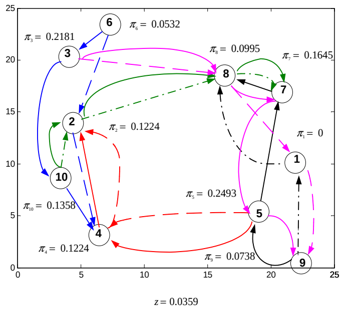
(b) Flow Set 1



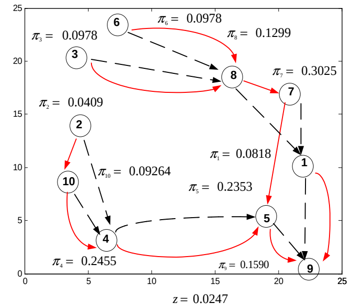
(c) Flow Set 2



(d) Flow Set 3

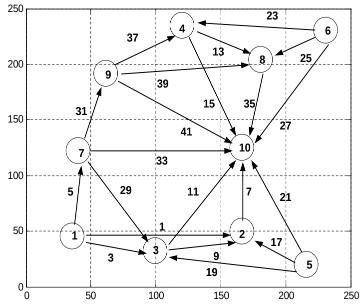


(e) Flow Set 4

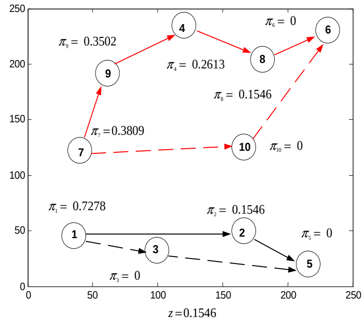


(f) Flow Set 5

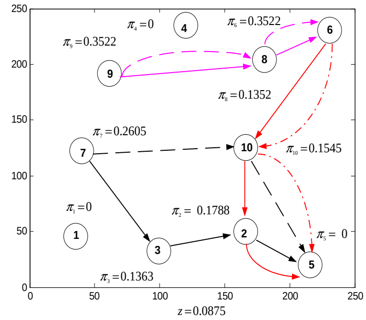
Fig. 2. Random network 1: optimal routing (solid lines) and MAC configurations and min-hop routing (dotted lines)



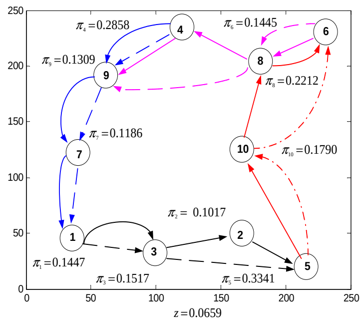
(a) Network 2



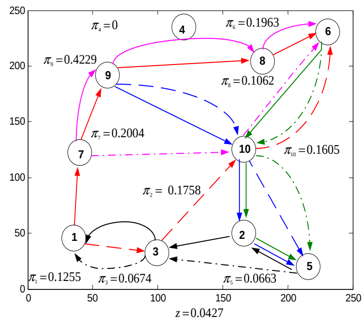
(b) Flow Set 1



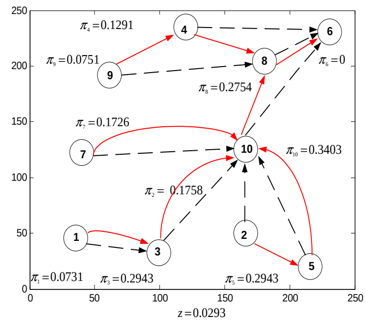
(c) Flow Set 2



(d) Flow Set 3



(e) Flow Set 4



(f) Flow Set 5

Fig. 3. Random network 2: optimal routing (solid lines) and MAC configurations and min-hop routing (dotted lines)

TABLE VI
PERFORMANCE GAIN OF THE JRM CONFIGURATION OVER DEFAULT CONFIGURATIONS

Network	Flow Set	JRM max-min throughput	Δ_1 max-min throughput	% Min gain	Δ_2 max-min throughput	% Min gain
1	1	0.1247–0.1249	0.0736	40.98	0.1125–0.1126	9.70
	2	0.1112–0.1115	0.0360	67.63	0.0602	45.86
	3	0.0494–0.0495	0.0312	36.84	0.0384–0.0385	22.06
	4	0.0359	0.0277	22.84	0.0293–0.0294	18.1
	5	0.0247	0.0107–0.0108	56.28	0.0116–0.0117	52.63
2	1	0.1543–0.1547	0.0766–0.0767	50.29	0.1381–0.1383	10.37
	2	0.0877–0.0879	0.0413–0.0414	52.79	0.0744–0.0745	15.05
	3	0.0686–0.0687	0.0547–0.0548	20.12	0.0576–0.0578	15.74
	4	0.0430-0.0431	0.0159–0.0160	62.79	0.0218–0.0220	48.84
	5	0.0294	0.0093–0.0095	67.68	0.0104	64.62

TABLE V
NUMERICAL VERSUS SIMULATION MAX-MIN THROUGHPUT

Network	Flow	Numerical	Simulation	% Diff
1	1	0.1227	0.1247–0.1249	1.79
	2	0.1107	0.1112–0.1115	0.72
	3	0.0493	0.0494–0.0495	0.41
	4	0.0359	0.0359	0
	5	0.0247	0.0247	0
2	1	0.1546	0.1543–0.1547	0.19
	2	0.0875	0.0877–0.0879	0.46
	3	0.0659	0.0686–0.0687	4.25
	4	0.0427	0.0430–0.0431	0.94
	5	0.0293	0.0294	0.34

is routed by a node, the node selects the flows with equal probability.

We have used two variants of these default configurations differing only by the value used for the attempt probability. In the default configuration 1, denoted Δ_1, we set the attempt probability to $1/N$ where N is the number of nodes in the network. In the default configuration 2, denoted Δ_2, the attempt probability is set to $1/N_a$, where N_a is the number of transmitting nodes in the network. If the routing is given, we can compute N_a easily and hence increase the attempt probabilities.

A. Routing and Selection Probability

We show in Figures 2 (b)–(f) and 3 (b)–(f) the optimal routing of each flow with solid lines and the optimal attempt probability of each node for the 10 scenarios. The computed max-min throughput of the flows is given at the bottom of each figure. Due to space limitation, we do not present the optimal values of $\boldsymbol{\tau}^*$. In each figure, we also present the routing for the two default configurations, indicated by dotted lines. Note that routing is the same for the two default configurations. We can see that the optimal attempt probabilities are very different from those of the default configurations and that, in most cases, minimum hop routing is not optimal. In particular, a node carrying high traffic and suffering high interference has a high attempt probability. From the optimal routing, we also note that most of the flows choose a path with high link quality. It is also interesting that, for all the scenarios, the optimal route of each flow is a single path. It means that splitting a flow to balance the load in a network does not seem to be a good solution for a random access network, since it increases collisions by increasing traffic in the competing nodes. A similar phenomenon is also observed in [6].

B. Throughput Gain

The max-min throughput for both default configurations is determined by simulation for each scenario and compared to the max-min throughput obtained by simulation by configuring the network using the JRM.

The interval of max-min throughput for the JRM, Δ_1, and Δ_2 configurations are shown in Table VI. The performance gain varies significantly from one scenario to another. The relative throughput gain ranges from 22.12% to 67.68% for the Δ_1 configuration and from 9.70% to 64.62% for Δ_2. From these results, we can conclude that the joint routing and MAC design provides an opportunity to significantly improve the max-min throughput of random access wireless networks.

VIII. CONCLUSIONS

In this paper, we study the joint configuration of routing and MAC parameters of a slotted ALOHA-based wireless network. We formulate a joint optimization problem that we solve for small networks of 10 nodes each and we validate by simulation the saturation assumption used in the analytical model. Via an extensive simulation campaign, we demonstrate that an optimal configuration has the following characteristics, at least in all the scenarios that we studied: (i) single path routing is optimal, (ii) most of the flows choose a path with high link quality instead of a minimum hop path, and (iii) the attempt probabilities of the nodes differ from each other significantly, where a node carrying high traffic and suffering high interference has a high attempt probability. We also determine the performance gain of the JRM configuration over the default configurations. The max-min throughput performance gain is found to be between 9.7% to 67.68% depending on the scenario. This work demonstrates that cross-layer design of routing and MAC yields significant improvement in throughput performance in fixed wireless networks using slotted ALOHA.

APPENDIX
A STATISTICAL TEST OF STABILITY

A. Methodology

The max-min throughput of a network is the maximum traffic rate that can be injected in each source such that the network will be stable. We consider that a network is stable if *all* its queues are stable. The problem is then to estimate whether a queue is stable for a given load. This is a complex problem for which we do not have a rigorous solution. Instead, we use a simple statistical test that can be justified as follows.

The test is based on the behavior of M/M/1/K queues (note that the same argument can be done using M/D/1/K queues). Recall that the loss probability P_K in an M/M/1/K is given by

$$P_K = \left(\frac{1-\rho}{1-\rho^{K+1}}\right)\rho^K \tag{18}$$

with queue utilization factor ρ. When K is large, if $\rho < 1$, we have

$$P_K \simeq (1-\rho)\rho^K \tag{19}$$

which is the standard formula for the M/M/1/∞ queue. This value will go to zero rather quickly as K gets large, so that the loss probability is very small unless ρ is very close to 1. If $\rho > 1$, we get for a large K

$$P_K \simeq (\rho-1)\frac{\rho^K}{\rho^{K+1}} = \frac{\rho-1}{\rho} \tag{20}$$

which is a pure fluid model. If $\rho = 1$, we get

$$P_K = \frac{1}{K+1}. \tag{21}$$

In other words, the buffer loss probability is a very powerful test for the stability of a queue. It gets close to 0 very quickly when $\rho < 1$ and increases reasonably fast when $\rho > 1$, as can be seen from part (a) of Fig. 4 for $K = 1000$.

To determine the stability of a network for a particular source rate, we consider that the buffer size of each queue is K instead of infinity, and assume that the system is unstable if P_K of any queue exceeds $1/(K+1)$. Increasing the source rate from a low value in several steps and checking the stability of each queue at each step by simulation, the maximum source rate yielding stability of all queues can be determined for a given network configuration.

B. Validation of the Test

Although the queues of a multi-hop slotted ALOHA network are not $M/M/1/K$, we assume that its packet loss behavior should be similar if the buffer size is set to a large value. We have verified this assumption as follows. The max-min throughput of each scenario is determined 10 times for increasing loads. The packet loss probabilities of all the queues for different source rates are plotted in Figures 4 (b), (c) and (d) for the Δ_1, Δ_2 and JRM configurations of flow set 5 of network 1. We see that in all cases P_K does increase from zero to a high value very quickly when the rate reaches a certain threshold, in the present case, within about 1% of the max-min throughput. Based on this, we can use the test with reasonable confidence that the error in estimating the maximum rate is not much more than 1%.

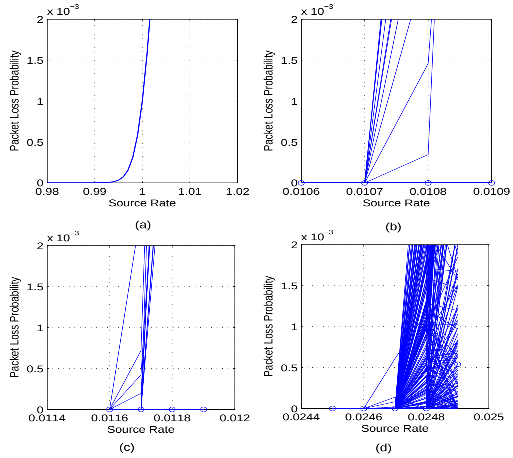


Fig. 4. Packet loss probability with source rate: (a) $M/M/1/1000$ queue, (b) Δ_1 configuration of network 1 for flow set 5, (c) Δ_2 configuration of network 1 for flow set 5, and (d) JRM configuration of network 1 for flow set 5

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