Delay Throughput Tradeoffs in Wireless Mesh Networks *(Invited)*

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***Abstract*—The support of delay-sensitive applications like VoIP, video conferencing, video streaming, etc. on scheduled mesh networks requires careful configuration of routing and scheduling. We formulate a delay-optimal joint routing and scheduling optimization problem that minimizes the maximum average delay perceived by any flow under the physical interference model.**

Due to the non-convex nature of this proposed problem, we cannot find the globally optimal joint routing and scheduling that minimizes the maximum per flow average delay. We overcome this issue by fixing the routing which makes the problem convex and then compute the optimal scheduling that results in minimum delay. We investigate several different routing strategies and compare the per flow average min-max delay when the scheduling is computed optimally. Based on these results, we provide engineering insights on configuring a wireless network for delay-sensitive applications.

***Index Terms*—Wireless mesh networks, scheduling, delay, throughput, non-linear optimization.**

I. Introduction

The deployment of wireless mesh networks is envisioned to provide anywhere and anytime low-cost connectivity in both developed and underdeveloped areas [1]. As the wireless medium is shared and scarce, there is a need to optimally design the network so as to extract the maximum performance from it. In other words, for a static wireless network supporting quasi-static traffic, the routing and scheduling have to be carefully configured.

Several delay-sensitive applications like VoIP, video conferencing, video streaming, etc. require that the network is optimized not just for throughput but also for delay. A significant part of the current literature is devoted to finding the optimal configuration (i.e., the routing and the scheduling) for extracting the maximum throughput from the network. In this paper, we show the need for optimizing for delay by showing that the naive approach based on optimizing for throughput does not necessarily result in solutions which also simultaneously have acceptable delay.

For a given static wireless mesh network and a given set of flows, assuming a simple delay model and the physical interference model, we propose a *non-linear* problem, that in principle computes the optimal joint routing and scheduling. The key difference between this model and the ones considered by Ramamurthi *et al.* [2] and by Birmiwal *et al.* [3] is the objective function. We consider minimizing the maximum delay perceived by any flow while they consider minimizing the average delay of all the flows taken together. The drawback of their approach is that some of the flows might unfairly receive large delays, even though the average delay of all the flows together is within acceptable limits. Another key difference is that, unlike those works, we investigate this problem from both the throughput and the delay perspectives and study configurations (routing and scheduling) that aim not only maximizing the throughput but also minimizing the delay.

Unfortunately, due to the non-convex nature of the model we propose, we cannot obtain the globally optimal joint routing and scheduling that minimizes the maximum delay. A similar non-convexity issue exists even in the models proposed in [2] and [3]. In our model, if the routing is fixed then our optimizing problem becomes convex. Thus, we can find the optimal scheduling that minimizes the maximum delay for a given routing. It is interesting to note that if we fix the scheduling, the model remains non-convex.

In this paper, we explore four different routing strategies. For each of these strategies, we configure the scheduling optimally in terms of min-max delay. The first one is the routing that gives the optimal max-min throughput obtained by solving the *joint routing and scheduling throughput optimal* (JRS-TO) linear program proposed by Karnik *et al.* [4]. This throughput-optimal routing combined with a delay-optimal scheduling performs well in terms of both the throughput and the delay. The second routing we consider is the single path (SP) routing that gives the largest max-min throughput and the third routing is the minimum hop routing (OMH) that gives the largest max-min throughput. The final routing we consider is obtained using the Dijkstra's algorithm for minimum hop paths (DMH). Depending upon the order in which different nodes are considered, we get different DMH routings. By design, the JRS-TO routing supports the maximum possible throughput for a given network at a given transmission power. The maximum throughput supported by each of the other three routings is in general much less than this global optimal throughput. This has a significant impact on the delay obtained with these routings at high loads as will be seen later. We present numerical results comparing the average delay performance of all the four different routings for 100 random realizations of 16-nodes networks and 20 random realizations of 25-nodes networks on a $20m \times 20m$ field and provide engineering insights into the design of delay-optimal networks.

In summary, our main contributions through this paper are as follows.

1) A non-linear program to compute the joint routing and scheduling that minimizes the maximum delay perceived by any flow.
2) A comparison of the delay performance of several heuristic methods to obtain good routings when the scheduling is computed optimally.
3) Engineering insights into the design of delay-optimal networks and how useful throughput optimal configurations are when delay is the performance criteria.

The rest of the paper is organized as follows. We give a review of the related work in Section II. In Section III, we define the problem and formulate the non-linear program that computes the optimal joint routing and scheduling that minimizes the maximum average delay. We then present several methods to select the routing strategies and find the corresponding delay optimal schedulings in Section IV and give several engineering insights on configuring a wireless network for delay-sensitive applications. We conclude in Section V with directions for future work.

II. Related Work

The capacity or the maximum sustainable throughput of a wireless network has been a subject of considerable interest in the literature. For a sample of these studies, see [4]–[8], etc. Each of these works has different goals, for instance, Gupta *et al.* [5] are interested in the asymptotic capacity of the network while Karnik *et al.* [4] are interested in explicitly finding the maximum sustainable throughput. They use different fairness criteria and different interference models. Some like [8] assume multiple channels of communication while [4], [5], [7] assume a single channel. Almost all the works have an underlying assumption of a quasi stationary traffic. Similarly, the optimization of delay has been considered in several works [2], [3], [9], [10]. While Florens *et al.* [9] and Gargano *et al.* [10] are interested in minimizing the number of time-slots required to collect a single packet of data from every node in the network at the gateway, Ramamurthi *et al.* [2] and Birmiwal *et al.* [3] are interested in minimizing the overall average delay when the arrival traffic follows a Poisson process.

The network and interference models assumed in this paper are similar to the ones used in [4] and [7]. We use a simple delay model for our FIFO queues which is akin to the one used in [2] which is based on the Kleinrock independence approximation [11] for a network of M/M/1 queues. While the delay model is simplistic, the results we obtain are very insightful. To the best of our knowledge, this is the first study that uses this delay model with min-max fairness criteria under the physical interference model. This is also the first work in the literature that combines a throughput optimal routing with a delay-optimal scheduling.

In the next section, we state our assumptions more precisely, describe the network and the interference models, and present the problem formulation.

III. Problem Formulation

A. Network Model

We consider a static wireless mesh network with quasi stationary channel gains. Assume that there are n nodes in this network, labeled as $1, 2, \ldots n$ and a set of flows, $\mathcal{F}$ with cardinality F. Every flow $f \in \mathcal{F}$ is defined as a triplet (f_s, f_d, λ_f), where f_s is the source of the flow, f_d is the destination and λ_f is the required throughput of the flow f. We model the delay on the links using a simple function based on the average delay expression for a M/M/1 queue and use the Kleinrock independence assumption [11] to compute the delay over a path composed of multiple links. It should be noted that there is a feasibility issue for any given $\mathcal{F}$ as it is possible that the network may not be able to support the required throughputs for the given set of flows.

We assume that there is no power control and every node transmits with the same power p. We also assume that there is a single channel of communication. We model the wireless interference using the physical model which is based on the Signal to Interference and Noise Ratio (SINR) [4], [5]. Experimental results [12] show that this models wireless interference more accurately than any other simpler model. Also, Iyer *et al.* [13] showed that the results obtained from using simpler models may be qualitatively different from those obtained from the physical model.

A directed wireless link from node i to node j is said to exist if

$$\frac{p_{i,j}}{N_0} \geq \beta \tag{1}$$

where $p_{i,j}$ is the total power received from node i at node j, N_0 is the thermal noise power and β is the minimum Signal to Noise Ratio (SNR) required for successful decoding of the message. If a link (i, j) is feasible, let c be the rate supported by it. This means that we assume a single rate model but extending it to a multi-rate model is straightforward by defining a link as a logical entity like in [7], instead of as a physical entity. Given a channel propagation model for $p_{i,j}$, we can compute the set of all feasible directed links $\mathcal{L}$ in the network using the SNR threshold condition given in (1). We make no restricting assumptions on the channel propagation model.

In a wireless network, even though all the links use the same channel, we can typically schedule a group of links to transmit at the same time without causing excessive interference to any intended receivers. We call such a subset of links an *independent set* (Iset). The interference model dictates which subset of links can be successfully activated at the same time. In the physical interference model, when two or more links are active on the same channel, every receiver considers the power from the transmitters other than its own as interference.

Under the physical interference model, a subset of feasible links, I, is an Iset only if they form a matching i.e.,

$$i \neq i' \wedge i \neq j' \wedge j \neq i' \wedge j \neq j' \quad \forall (i,j),(i',j') \in I \tag{2}$$

and all the corresponding receivers have an SINR greater than or equal to β, i.e., for all the links (i,j) in the Iset I,

$$p_{i,j} \geq \beta \left[N_0 + \sum_{(i',j') \in I \setminus \{(i,j)\}} p_{i',j} \right]. \tag{3}$$

The sum in (3) is the total interference received by the destination node j of link (i,j) due to the transmissions on all the other links (i',j') in I. We compute all the Isets compatible with constraints (2) and (3). Let this set of Isets be $\mathcal{I}$. Next, we state the problem we consider in this paper.

Problem Statement: Given a network with n nodes, a transmission power p and a set of F flows with a required throughput of λ_f for flow f, find the optimal joint routing and scheduling that minimizes the average delay perceived by any flow (assuming a simple delay function), if the network can support the given throughputs of all the flows.

This problem statement can be readily translated into a non-linear program which we present in the next sub-section.

B. The Non-Linear Program

Recall that we model the delay on the links using a simple function based on the average delay expression for a M/M/1 queue and use the Kleinrock independence assumption [11] to compute the delay over a path composed of multiple links. We know that for a M/M/1 queue with an arrival rate of λ and a mean service time of $1/\mu$, the average delay in the queue is given by

$$\delta = \frac{1}{\mu - \lambda} \tag{4}$$

The mean service time is typically inversely proportional to the link capacity. In a wireless network, the capacity of a link is not straightforward. In a scheduled network, it depends on the amount of time the link is scheduled. Thus, if $x^f_{i,j}$ is the amount of traffic of flow f on link (i,j) and α_k is the fraction of time the Iset I_k is active for, then using equation (4), the average delay experienced by any flow on link (i,j) is given by

$$\frac{1}{c \sum\limits_{(i,j) \in I_k} \alpha_k - \sum\limits_{f \in \mathcal{F}} x^f_{i,j}} \tag{5}$$

Thus, the contribution to the average delay of a flow f due to $x^f_{i,j}$ amount of flow on link (i,j) is given by

$$\frac{x^f_{i,j}/\lambda_f}{c \sum\limits_{(i,j) \in I_k} \alpha_k - \sum\limits_{f \in \mathcal{F}} x^f_{i,j}} \tag{6}$$

Summing the contributions given in equation (6) over all the links gives the average delay experienced by the flow f, i.e.,

$$\delta_f = \sum_{(i,j) \in \mathcal{L}} \frac{x^f_{i,j}/\lambda_f}{c \sum\limits_{(i,j) \in I_k} \alpha_k - \sum\limits_{f \in \mathcal{F}} x^f_{i,j}} \tag{7}$$

Using this closed form expression for delay and given the set of nodes, the set of flows $\mathcal{F}$, the set of links $\mathcal{L}$ and the set of Isets $\mathcal{I}$, the non-linear program to compute the optimal joint routing and scheduling that minimizes the maximum average delay perceived by any flow, is formulated as follows.

$$\mathcal{P}: \qquad \underset{\mathbf{x},\boldsymbol{\alpha}}{\text{Minimize}}\ \delta$$

subject to

$$\delta \geq \frac{1}{\lambda_f} \sum_{(i,j) \in \mathcal{L}} \frac{x^f_{i,j}}{c \sum\limits_{(i,j) \in I_k} \alpha_k - \sum\limits_{f \in \mathcal{F}} x^f_{i,j}} \ \forall f \tag{8}$$

$$\sum_{(i,j) \in \mathcal{L}} x^f_{i,j} - \sum_{(j,i) \in \mathcal{L}} x^f_{j,i} = \begin{cases} \lambda_f & \text{if } i = f_s \\ -\lambda_f & \text{if } i = f_d \\ 0 & \text{otherwise} \end{cases} \ \forall i,\ \forall f \tag{9}$$

$$\sum_f x^f_{i,j} \leq c \sum_{(i,j) \in I_k} \alpha_k \ \forall (i,j) \tag{10}$$

$$\sum_{I_k \in \mathcal{I}} \alpha_k \leq 1 \tag{11}$$

Constraint (8) represents the min-max objective for the delay. Flow conservation at all the nodes leads to constraint (9) while the link capacity constraint results in (10). Constraint (11) ensures that the sum of the fractions of time all Isets are activated for is less than or equal to 1.

It is not difficult to verify using the Hessian matrix that constraint (8) is non-convex if both $x^f_{i,j}$ and α_k are variables. We can also see that it is convex in α_k if we assume $x^f_{i,j}$ to be given but still non-convex in $x^f_{i,j}$ if we assume α_k to be given. In other words, this implies that if the routing is given, i.e., the $x^f_{i,j}$'s are given, then we can compute the globally optimal schedule that results in min-max delay. This is the approach we take, i.e., we will fix the routing strategy and compute the delay optimal scheduling for that routing. Hence, we need to find effective methods to fix the routing.

In the next sub-section, we discuss the throughput-optimal problem posed by Karnik *et al.* [4] which is the key to the methods we propose to select the routing strategy.

C. The Throughput-optimal Problem

Recall that in the delay-optimal problem $\mathcal{P}$, a set of flows along with the required throughputs are given. This given set of flows may or may not be feasible in the given network at the given transmission power. On the other hand, in the throughput-optimal problem (call it $\mathcal{P}_T$), the objective is to greedily maximize the minimum throughput for a given set of flows, $\mathcal{F}'$ given without any throughput requirement. Thus, unlike in the delay-optimal problem, there is no issue of feasibility for the given set of flows, $\mathcal{F}'$ in the throughput optimal problem. In a useful variation of this problem $\mathcal{P}_T$, we can a priori assign different weights to different flows in $\mathcal{F}'$. Using this idea, we can relate the set of flows $\mathcal{F}$ and their required throughputs in $\mathcal{P}$ with the set of flows $\mathcal{F}'$ and their corresponding weights in $\mathcal{P}_T$. In $\mathcal{F}$, let λ_{min} be the required throughput of the flow with the minimum throughput requirement. Using this quantity, we compute the weight of every flow f in $\mathcal{F}$ as $w_f = \lambda_f/\lambda_{\text{min}}$ and define the set of flows $\mathcal{F}'$ as all the flows in $\mathcal{F}$ with these weights.

Since computing the global optimal solution to the non-linear program $\mathcal{P}$ is not possible, we use $\mathcal{P}_T$ to infer the feasibility of the given flow set $\mathcal{F}$ using the set $\mathcal{F}'$ with the corresponding weights. We also hope that the routing it produces is effective in achieving low delays. Given the set of nodes, the transmit power p, the set of links $\mathcal{L}$, the set of flows $\mathcal{F}'$, the corresponding weights w_f's and the set of Isets $\mathcal{I}$, the throughput-optimal problem $\mathcal{P}_T$ (given in [4]) is as follows.

$$\mathcal{P}_T : \qquad \underset{\boldsymbol{\alpha},\mathbf{y}}{\text{Maximize}}\ \lambda \tag{12}$$

$$\sum_j x^f_{i,j} - \sum_j x^f_{j,i} = \begin{cases} w_f\lambda & \text{if } i = f_s \\ -w_f\lambda & \text{if } i = f_d \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

$$\sum_{f=1}^{n} x^f_{i,j} \leq c \sum_{(i,j)\in I_k} \alpha_k \quad \forall (i,j) \in \mathcal{L} \tag{14}$$

$$\sum_{I_k\in\mathcal{I}} \alpha_k \leq 1 \tag{15}$$

Let λ^p_{to} be the optimal solution to $\mathcal{P}_T$ for a given transmit power p. The given set of flows $\mathcal{F}$ is feasible if and only if $\lambda^p_{\text{to}} \geq \lambda_{\text{min}}$. $\mathcal{P}_T$ not only determines the feasibility of the given set of flow throughputs for $\mathcal{P}$ but also provides a method to choose a routing strategy (we discuss this in detail in the next subsection). We call this the **JRS-TO** routing.

In the next sub-section, we discuss all the four different methods (including JRS-TO) we have explored to select the routing strategies and how we determine the routing variables $x^f_{i,j}$'s for each of them.

D. The Different Routing Strategies

The goal of fixing the routing is to determine the $x^f_{i,j}$'s so that we can solve the delay-optimal problem $\mathcal{P}$ and find the delay-optimal scheduling for the given routing including the min-max delay for that routing. In the following, we assume that the set of flows $\mathcal{F}'$ and their corresponding weights w_f's for $\mathcal{P}_T$, are calculated from the given set of flows $\mathcal{F}$ and their required throughputs λ_f's using the technique given in the previous sub-section.

Our simplest method to select the routing strategy is based on the Dijkstra's algorithm for finding the minimum hop routing for a given transmit power p (short form, **DMH** routing). It results in a randomly chosen min-hop routing depending on the order in which the nodes are considered in the Dijkstra's algorithm. Obtaining the DMH routing is the least computational intensive of all the routing strategies considered.

Once we obtain the DMH routing which is a single path routing, we solve $\mathcal{P}_T$ using only the links in this routing. Let λ^p_{dmh} be the optimal solution of this problem at the given transmit power p. Then, the given required throughputs for the set of flows $\mathcal{F}$ are feasible on this routing if and only if $\lambda^p_{\text{dmh}} \geq \lambda_{\text{min}}$. However, the resultant $x^f_{i,j}$'s correspond to the max-min throughput of λ^p_{dmh} and need to be scaled according to the required throughputs. Thus, the $x^f_{i,j}$'s for $\mathcal{P}$ can be determined by scaling the $x^f_{i,j}$'s obtained to achieve λ^p_{dmh} by multiplying them by $\lambda_{\text{min}}/\lambda^p_{\text{dmh}}$.

The second strategy is the **JRS-TO** routing described in the previous sub-section. Let λ^p_{to} be the optimal solution to $\mathcal{P}_T$ for the given transmit power p. This is the global optimal achievable throughput for the given weights which means that if $\lambda^p_{\text{to}} < \lambda_{\text{min}}$, then there exists no routing for which the given flow throughputs are feasible. On the other hand, if $\lambda^p_{\text{to}} \geq \lambda_{\text{min}}$, then it implies that the given set of flows $\mathcal{F}$ and their required throughputs are feasible. However, the resultant $x^f_{i,j}$'s correspond to the max-min throughput of λ^p_{to} and need to be scaled according to the required throughputs. Thus, the $x^f_{i,j}$'s for $\mathcal{P}$ are determined by scaling the $x^f_{i,j}$'s in the solution to $\mathcal{P}_T$ by multiplying them by $\lambda_{\text{min}}/\lambda^p_{\text{to}}$. The next two strategies are based on solving variants of $\mathcal{P}_T$.

The third strategy involves imposing a single path routing (short form, **SP** routing) on $\mathcal{P}_T$. To obtain this routing, we have to define a new set of binary variables $w_{i,j}$ that indicate whether the link (i,j) is used or not and add the following new constraints to $\mathcal{P}_T$.

$$x^f_{i,j} \leq w_{i,j} \quad \forall (i,j) \quad \forall f \tag{16}$$

$$\sum_j w_{i,j} \leq 1 \quad \forall i, \quad w_{i,j} \in \{0,1\} \tag{17}$$

The solution to this new problem which is a binary program gives the SP routing. Let λ^p_{sp} be the optimal solution to this problem. In general, this value is as good as the global optimal solution λ^p_{to} as discussed in [7]. The given required throughputs are feasible on this routing if and only if $\lambda^p_{\text{sp}} \geq \lambda_{\text{min}}$. And if it is feasible, then the $x^f_{i,j}$'s for $\mathcal{P}$ are determined by re-scaling the $x^f_{i,j}$'s in the solution to this new problem by multiplying them by $\lambda_{\text{min}}/\lambda^p_{\text{sp}}$.

The fourth and the final strategy is based on a minimum hop variation of the SP routing, i.e., we are looking for a min-hop routing that yields the largest max-min throughput. We call this the **OMH** routing. Let d_f be the minimum number of hops from the source f_s to the destination f_d for the flow f (this can be computed using the Dijkstra's algorithm). We define a new problem by adding the following constraint, in addition to the constraints for SP routing (constraints (16) and (17)), to $\mathcal{P}_T$.

$$\sum_{i,j} x^f_{i,j} \leq \lambda_f d_f \tag{18}$$

Let λ^p_{omh} be the optimal solution to this problem. This is the largest possible throughput for any minimum hop routing. In general, this throughput is much less than the global optimal solution λ^p_{to} [7]. Again, the given required throughputs are feasible on this routing if and only if $\lambda^p_{\text{omh}} \geq \lambda_{\text{min}}$ and if feasible, the $x^f_{i,j}$'s for $\mathcal{P}$ are determined by scaling the $x^f_{i,j}$'s in the solution to this new problem by multiplying them by $\lambda_{\text{min}}/\lambda^p_{\text{omh}}$.

Note that all the routing strategies considered, except the JRS-TO routing are single-path. Also note that, because we have

$$\lambda^p_{\text{to}} \geq \lambda^p_{\text{sp}} \geq \lambda^p_{\text{omh}} \geq \lambda^p_{\text{dmh}} \tag{19}$$

a given set of flows and throughput requirements, i.e., $\mathcal{F}$ may not be feasible on all the routings.

For each of the routings at the given transmit power p, using the scaled $x^f_{i,j}$'s determined for the required throughputs, we compute the scheduling (α_k's) that minimizes the maximum average delay for that given routing by solving $\mathcal{P}$. Note that the flow constraint (9) in $\mathcal{P}$ is redundant when the routing ($x^f_{i,j}$'s) is given. In this case, the solution to $\mathcal{P}$ gives a delay optimal scheduling along with the selected routing.

In the next section, we present numerical results for 100 random realizations of 16-nodes networks and 20 random realizations of 25-nodes networks, and provide engineering insights into the design of delay-optimal networks.

IV. Numerical Results and Engineering Insights

We have generated 100 random realizations of 16-nodes networks and 20 random realizations of 25-nodes networks on a 20×20 field. The network in Figure 1, labelled as netA is one of the realizations of a 16-nodes network. For the sake of numerical results, we have assumed that there is a traffic pattern converging towards the special node at the center of the field called the sink or the gateway. More specifically, we have assumed that there is a flow from every node to the sink and that all the flows have the same throughput requirement, i.e., $\lambda_f = \text{constant}\ \forall f$ and hence all the corresponding flows in $\mathcal{P}_T$ have the same weights. We have also assumed a unit rate on all the feasible links, i.e., $c = 1$ and used the network parameters given in Table I with the following channel propagation model.

$$P_{i,j} = \frac{G_{i,j} P}{(\frac{d_{i,j}}{d_0})^\eta}$$

$$\text{SNR} = \frac{P_{i,j}}{N_0}$$

where $d_{i,j}$ is the physical distance between nodes i and j, $G_{i,j}$ is the channel gain on link (i,j) that accounts for channel fading and shadowing, d_0 is the near-field cross over distance, η is the path-loss exponent and N_0 is the thermal noise power in the frequency band of operation. Recall that we assume the channel gains to be quasi time-invariant. For simplicity in the numerical calculations, we have assumed the same constant G on all the links. However, this should not be construed as a limitation as computations with different G on different links are not more complex.

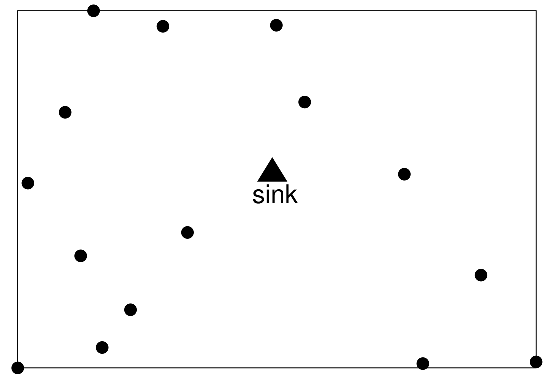


Fig. 1: A Random 16 Nodes Network: netA

β	6.4 dB
N_0	-100 dBm
η	3
d_0	0.1 m
G_l	1

TABLE I: Network parameters used for obtaining numerical results

Let λ^p_* be the solution to the linear program $\mathcal{P}_T$ for a given transmit power p. If λ is the required throughput for every flow and $\lambda \leq \lambda^p_*$, then the load (ρ) on the network is defined as

$$\rho = \frac{\lambda}{\lambda^p_*}$$

Note that ρ is also a function of p and a given λ could correspond to different ρ's at different powers. In addition to the JRS-TO routing, the optimal solution to $\mathcal{P}_T$ also has a scheduling, which we call the throughput optimal (TO) scheduling. Using this scheduling, we can compute the default delay for a given load ρ as follows. First scale the optimal routing values $x^f_{i,j}$ by multiplying them by ρ. Next, use these scaled $x^f_{i,j}$'s and the TO scheduling α_k's to compute the default delay using equation (7).

If λ^p_r is the maximum throughput supported by the routing r at the given transmit power p (which is $\leq \lambda^p_*$), then for a given load ρ, we scale the routing variables $x^f_{i,j}$'s by multiplying them with $\rho\lambda^p_*/\lambda^p_r$. Thus, for a given network at a given transmission power, the load factor ρ is computed with respect to λ^p_* while the $x_{i,j}$'s are scaled with respect to λ^p_r. To determine the min-max delay at a given load ρ for a given routing r at the given transmit power p, we recompute the scheduling by solving $\mathcal{P}$ with the scaled $x_{i,j}$'s.

For a given network at the given transmit power, we compared the default delay obtained by using the throughput optimal routing and scheduling with the delay obtained by using each of the routing strategies (JRS-TO, SP, OMH and DMH) with their associated delay optimal scheduling. For the random 16-nodes network, netA given in Figure 1, we have plotted a comparison of these various delays versus the load

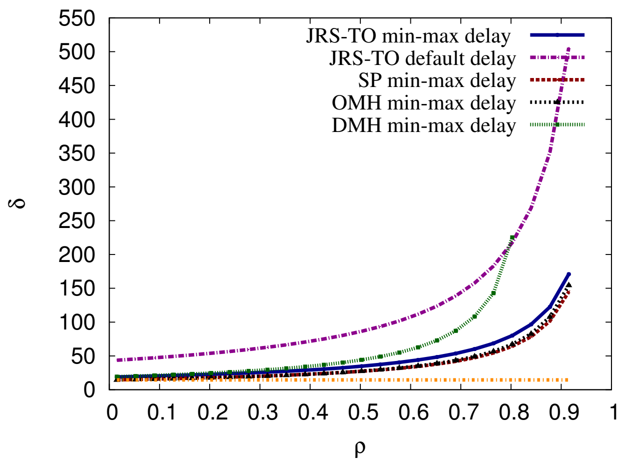


Fig. 2: Comparison of delays due of different routing strategies versus load at $p = -31.41$ dBm for netA

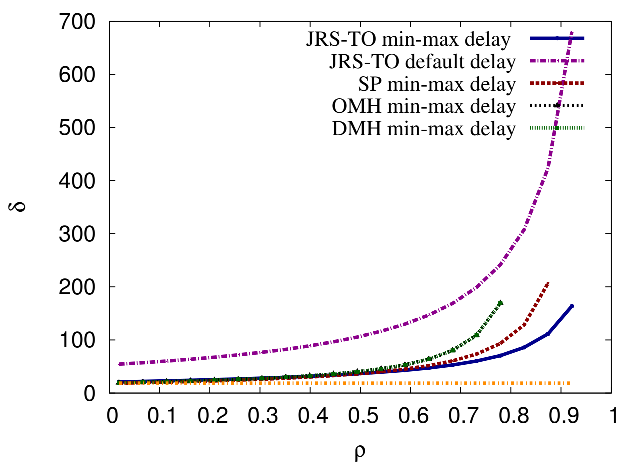


Fig. 3: Comparison of delays due of different routing strategies versus load at $p = -37.23$ dBm for netA

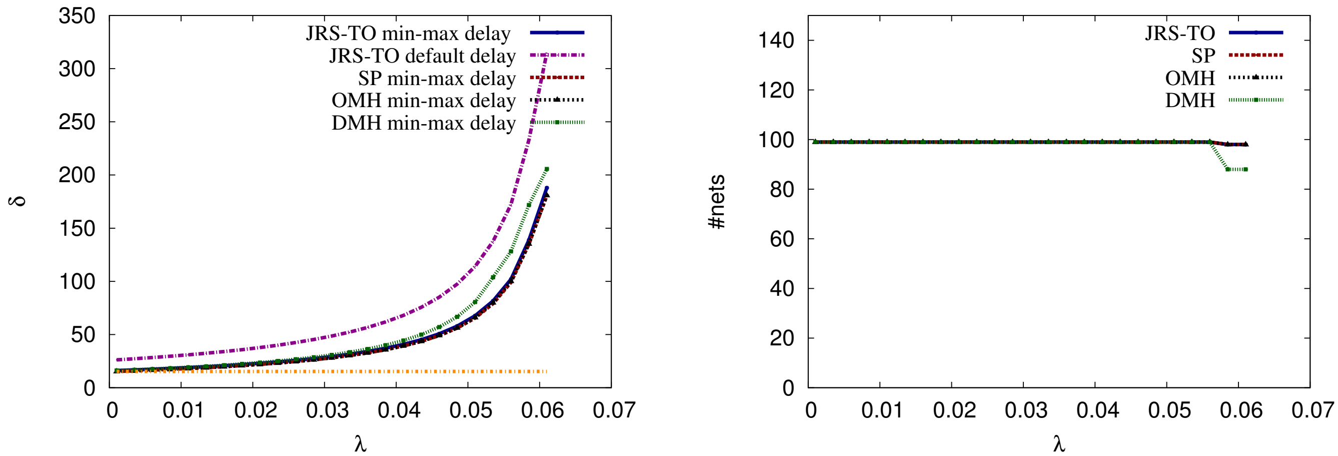
at a power of -31.41 dBm in Figure 2 and at a power of -37.23 dBm in Figure 3. From these plots, we note that there is a significant difference (in the range of two to four times) between the default delay of JRS-TO and the lowest of the min-max delay obtained with optimal scheduling. This means that optimizing the scheduling of a network for delay certainly makes a large improvement in the performance of delay-sensitive applications. We also note that the delay in DMH routing increases exponentially when the load approaches 0.8. This is because the maximum throughput achievable by using DMH routing is only around 0.8 times the global optimal throughput. We also observe that, when the load gets close to 1, the other three routings also experience large absolute delays. The reason for this is that when the network is operated close to its maximum capacity, the utilization of one or more links gets close to 1 as well, leading to large delays.

The plots in Figures 2 and 3 also suggest that there are three different regimes of operation, viz., low load, medium load and high load. When $\rho < 0.5$, we are in the low load regime while $\rho \geq 0.8$ is considered the high load regime. The one between these two is the medium load regime. At low loads, we observe that almost all the routing strategies we have considered, have similar delay performances. However, in the high load regime, we clearly cannot rely on a minimum hop routing as this routing fails to even support these high throughputs. Thus, at high loads, we have to first find a throughput optimal routing and then optimize the scheduling for the best delay performance. In the medium load regime, we observe that the delay performance of the minimum hop routing degrades very fast, mandating again the use of a throughput optimal routing. The reason for this is that as the load on the network approaches the maximum throughput supported by a minimum hop routing (which is significantly lower than the optimal throughput), the utilization of one or more links approaches 1 which leads to a rapid degradation of the overall delay performance.

For each of the 100 random realizations of 16-nodes networks and the 20 random realizations of 25-nodes networks, we see plots similar to the ones in Figures 2 and 3. As different loads in different networks correspond to different throughputs, it is not meaningful to average the delay of all the 100 networks at a given load. We, instead, average their delay at a given throughput and in Figures 4, 5 and 6, we plot the average delay versus a given throughput for 100 random realizations of 16-nodes networks at three different powers. In each of these figures, the first sub-figure gives a plot of delay vs throughput while the second sub-figure plots the number of networks supporting the given throughput. Similarly, in Figures 7 and 8, we plot the average delay versus throughput for 20 random realizations of 25-nodes networks for JRS-TO and DMH routing strategies at two different powers. For 25-nodes networks, we do not compute for the SP and the OMH routing strategies, as it involves solving large mixed integer programs which require large computations times.

As throughput gets high, the number of networks that are able to support such throughputs decreases. The abrupt jumps in the delays at higher throughputs occur because some of the networks are no longer able to support the higher throughput and were responsible for a high delay just before the jump. These plots (in Figures 4-8) confirm our earlier claims, that there is a need to optimize the scheduling for JRS-TO routing (because the min-max delay is significantly lower than the default delay), the adequacy of the DMH routing at low throughputs or equivalently at low loads (because all the routing strategies have a similar delay performance in this load regime) and the need for using the JRS-TO routing to support a high throughput and a low delay (notice the decrease in number of networks supporting high throughputs for different routing strategies).

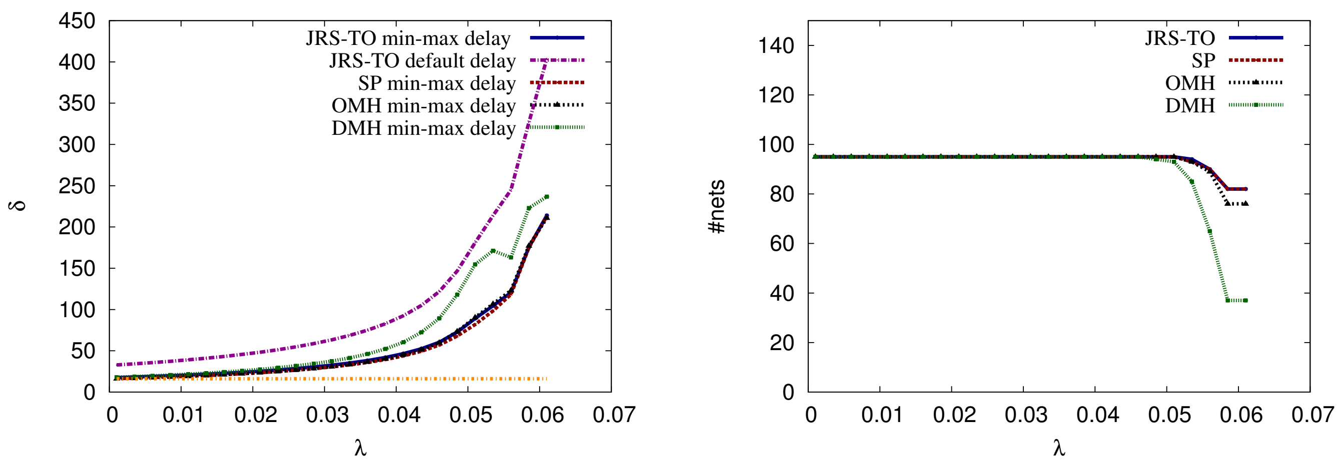
Focusing on the low load case, in Figure 9, we have plotted the average delay vs the transmit power for all the four routing strategies for 100 random 16-nodes networks at a very low throughput of $\lambda = 0.001$. As expected, we note an overall decreasing trend in the average delay as the transmission



(a) Delays vs Throughput

(b) Number of networks supporting a given throughput

Fig. 4: Comparison of average delays for various routing strategies at $p = -31.41$ dBm for 100 random 16-nodes networks



(a) Delays vs Throughput

(b) Number of networks supporting a given throughput

Fig. 5: Comparison of average delays for various routing strategies at $p = -33.74$ dBm for 100 random 16-nodes networks

power increases. We also observe that this plot suggests that on an average the OMH routing is better than all the other routing strategies considered. However, in general, for a given network, we found that no routing strategy is clearly better than all the other routing strategies at all the transmission powers. At low loads, if a random access protocol is used, we expect the average delay to be proportional to the diameter (the number of hops in the longest min-hop path) of the network (which is a function of the transmit power). But, because we use fixed scheduling, even at low loads, the average delay is quite high, in the range of n to $2n$, where n is the number of nodes in the network.

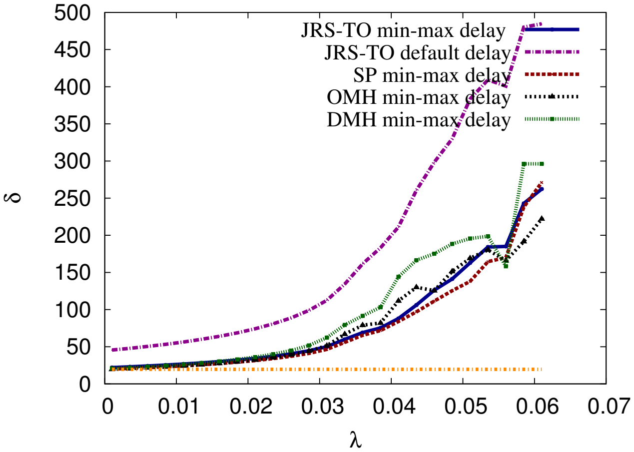
In summary, we learn three important lessons from these numerical results. They are

1) There is a need to optimize the scheduling for obtaining better delay performance.
2) We cannot rely on a minimum hop routing if we require a high throughput and low delay for an application.
3) At low loads, a min-hop routing achieves an acceptable delay performance.

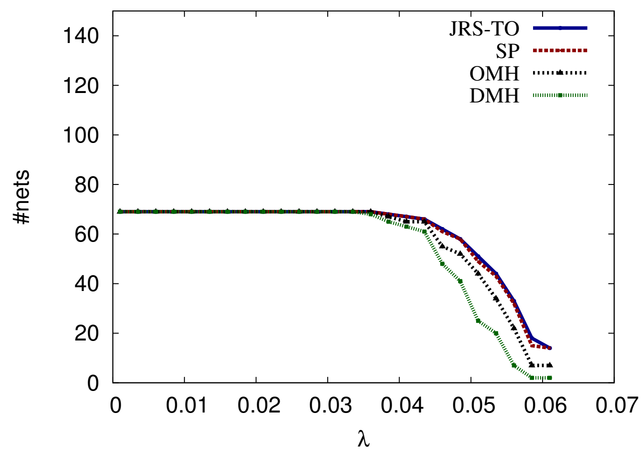
V. Conclusion and Future Work

In conclusion, this study shows that there is a need to optimize the scheduling for minimizing delay because the default delay due to the joint routing and scheduling computed for maximum throughput can be quite large. We also observed that we cannot rely on a simple routing like minimum hop if a high throughput and low delay network performance are required.

Exploration of other routing strategies and different flow patterns remains part of our future work.

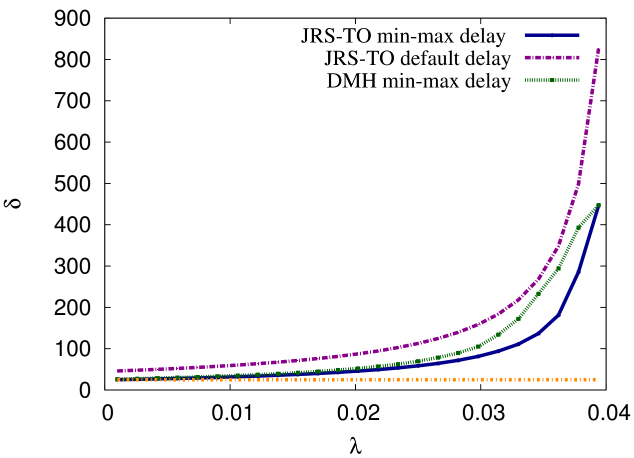


(a) Delays vs Throughput

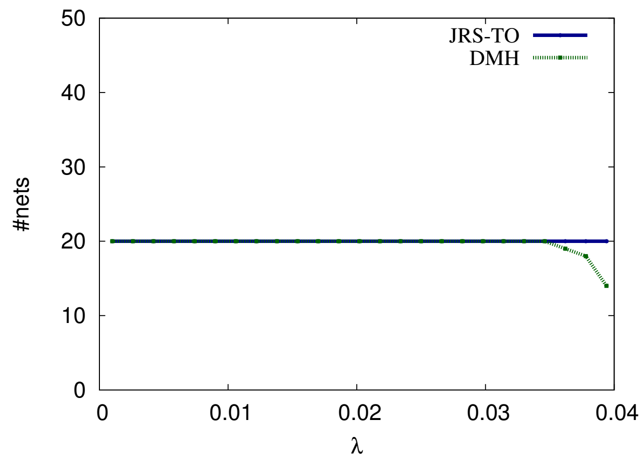


(b) Number of networks supporting a given throughput

Fig. 6: Comparison of average delays for various routing strategies at $p = -37.23$ dBm for 100 random 16-nodes networks



(a) Delays vs Throughput



(b) Number of networks supporting a given throughput

Fig. 7: Comparison of average delays for various routing strategies at $p = -31.41$ dBm for 20 random 25-nodes networks

References

[1] P. Pathak and R. Dutta, "A survey of network design problems and joint design approaches in wireless mesh networks," *IEEE Communications Surveys and Tutorials*, vol. 13, no. 3, pp. 396–428, Sep. 2011.

[2] V. Ramamurthi, A. Reaz, and B. Mukherjee, "Optimal capacity allocation in wireless mesh networks," in *IEEE Global Telecommunications Conference (Globecom)*, Dec. 2008, pp. 1–5.

[3] S. Birmiwal, J. Nair, D. Manjunath, and R. R. Mazumdar, "Delay minimization in multihop wireless networks: Static scheduling does it," in *Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), 10th International Symposium on*, May 2012, pp. 97–102.

[4] A. Karnik, A. Iyer, and C. Rosenberg, "Throughput-optimal configuration of fixed wireless networks," *IEEE/ACM Transactions on Networking*, vol. 16, no. 5, pp. 1161–1174, 2008.

[5] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[6] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *Automatic Control, IEEE Transactions on*, vol. 37, no. 12, pp. 1936–1948, Dec 1992.

[7] J. Luo, C. Rosenberg, and A. Girard, "Engineering wireless mesh networks: Joint scheduling, routing, power control and rate adaptation," *IEEE/ACM Transactions on Networking*, vol. 18, no. 3, pp. 1387–1400, Oct. 2010.

[8] A. Ghosh, O. D. Incel, V. S. A. Kumar, and B. Krishnamachari, "Multichannel scheduling and spanning trees: Throughput; delay tradeoff for fast data collection in sensor networks," *Networking, IEEE/ACM Transactions on*, no. 99, 2011.

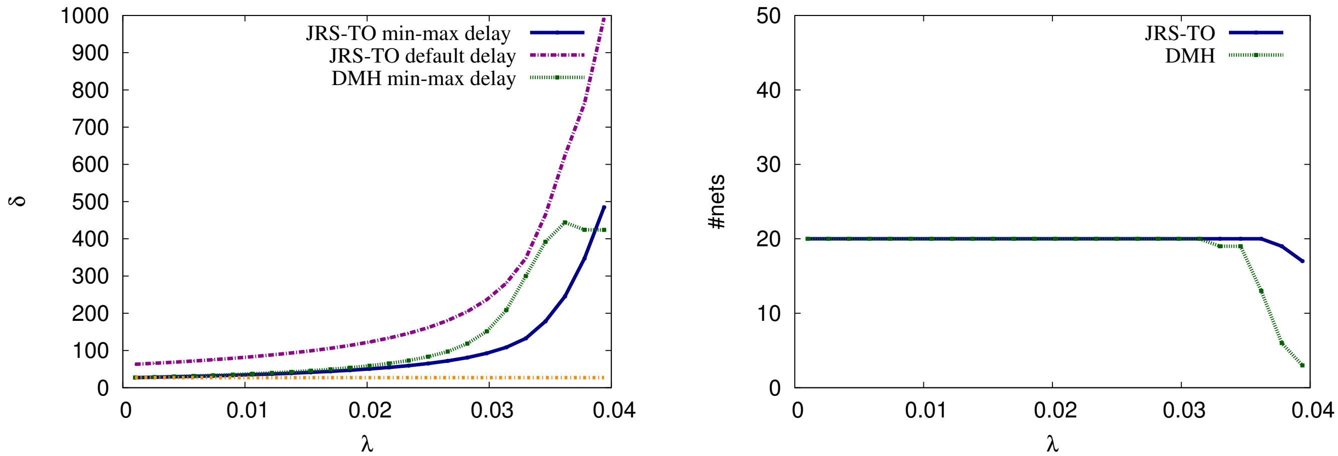
[9] C. Florens, M. Franceschetti, and R. McEliece, "Lower bounds on data collection time in sensory networks," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1110–1120, Aug. 2004.

[10] L. Gargano and A. A. Rescigno, "Optimally fast data gathering in sensor networks," *Mathematical Foundations of Computer Science*, vol. 4162/2006, pp. 399–411, 2006.

[11] L. Kleinrock, *Communication Nets: Stochastic Message Flow and Delay*. New York: Macgraw-Hill, 1964.

[12] R. Maheshwari, J. Cao, and S. R. Das, "Physical interference modeling for transmission scheduling on commodity wifi hardware," *IEEE INFOCOM*, Apr. 2009.

[13] A. Iyer, C. Rosenberg, and A. Karnik, "What is the right model for wireless channel interference?" *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2662–2671, 2009.



(a) Delays vs Throughput

(b) Number of networks supporting a given throughput

Fig. 8: Comparison of average delays for various routing strategies at $p = -33.74$ dBm for 20 random 25-nodes networks

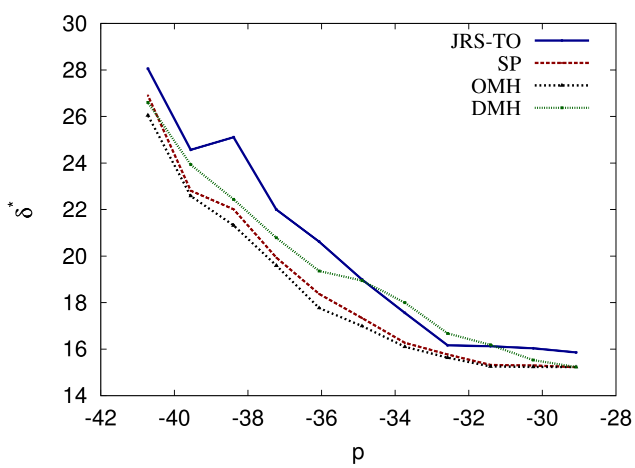


Fig. 9: Comparison of the average delays vs power for various routing strategies for 100 random 16-nodes networks at a very low throughput of $\lambda = 0.001$