

# On the Capacity and Scheduling of a Multi-Sector Cell with Co-Channel Interference Knowledge

Patrick Mitran, Catherine Rosenberg, John Sydor, Jun Luo, and Samat Shabdanov

**Abstract**—We study the capacity of a single-hop wireless cell comprised of  $M$  fixed directional antennas concentrically arranged around a common axis and each reusing the same channel. The premise for this architecture is that co-channel interference between the fixed directional beams can be identified and reported, allowing joint conflict-free scheduling, i.e., it becomes possible to compute the schedule over all the  $M$  sectors together. We are primarily interested in studying the capacity gains that can be realized as a function of the number of sectors, the parameters of the directional antennas and other system parameters. Our analytical and numerical results yield several useful engineering insights on the selection of antenna parameters, the impact of node placement, and the impact of physical layer parameters. This study demonstrates how Space Division Multiple Access (SDMA) can yield excellent capacity provided inter-cell co-channel coexistence is achieved. Such coexistence techniques are now being considered for cognitive radio TV White Space systems and by evolving standards such as the IEEE 802.16h.

**Index Terms**—Sectoring, Spatial Reuse, Scheduling, Cognitive Radio, IEEE 802.22, IEEE 802.16h.

## I. INTRODUCTION

Advanced metropolitan wireless networks capable of high capacity are being proposed using a variety of approaches. Several fundamental methods have been proposed to improve the aggregate access rate, including near Shannon limit error control coding [8], diversity and spatial multiplexing techniques, including MIMO [12], as well as multicarrier modulation [2]. Some approaches, such as MIMO, are deployment sensitive and show diminished capacity as the multipath environment loses its richness or has dimensional or placement constraints on handset antennas. The operation of cellular radio in sub-urban cell-edge locations or the use of MIMO for TV band applications are deployment scenarios limited by such constraints. However, such difficult scenarios can be addressed by the space division multiple access (SDMA) approach addressed in this paper, where directional antennas are used to sub-divide the cell into sectors that reuse bandwidth. The advantages of such an approach has been noted before [7], [11] and in principle it can be relatively simple to implement since it does not require any major modification to end user devices and/or has changes mostly constrained to the media access control layer of the wireless network. However, careful control of co-channel interference in such scenarios is required as is the need for efficient and fast scheduling, especially in time varying fading environments typical of mobile wireless.

P. Mitran, C. Rosenberg and S. Shabdanov are with the Department of Electrical and Computer Engineering, University of Waterloo, Canada. J. Luo is with the School of Computer Engineering, Nanyang Technological University, Singapore. J. Sydor is with Communications Research Centre, Ottawa, Canada.

Interference identification and interference-free scheduling approaches have been identified in formative cognitive and co-channel cellular coexistence standards such as the IEEE 802.22 and IEEE 802.16h [13]. With such standards being on the verge of implementation, the SDMA approaches addressed in this paper can be considered to supplement the bandwidth enhancing methods identified above.

In this paper we are interested in studying the performance that can be achieved in a single-hop static system having both a base station equipped with several fixed directional antennas and information about the co-channel interference created at all nodes. We assume that the aggregate link gains, including fading and shadowing between all transmitters and receivers are known, and that the resulting systemic co-channel interference of all receivers in the cell can be measured and transmitted to the base station. Thus the base station can compute the co-channel interference at every user for any schedule. In particular, in this study, we restrict ourselves to a single point-to-multipoint cell and study the capacity achievable when the base station is equipped with  $M$  directional antennas that create multiple sectors and joint conflict-free scheduling based on co-channel interference is performed (where “joint” refers to the fact that we compute the schedules over all the  $M$  sectors together). We use the term “capacity” in a broad sense now. We will define it more precisely later. We are primarily interested in studying capacity gains that can be realized as a function of the number of sectors, the parameters of the directional antennas and other system parameters.

The contributions of this paper are four-fold:

- We formulate a joint offline scheduling problem that takes co-channel interference into account to compute the system capacity defined with respect to a max-min throughput objective. This results in a very large linear program that cannot be solved by simple techniques for realistic scenarios. The antenna model and the fading model can be as sophisticated as one needs.
- We develop exact numerical tools that allows us to compute the capacity for medium-size systems and approximate tools for large systems. These computational tools work for any antenna and fading models.
- We propose an analytical upper bound on the capacity under some assumptions.
- We illustrate the gain in capacity that sectoring can bring and the impact of some critical system parameters on this capacity for a realistic antenna model.

This study can be seen as providing the critical benchmarks necessary when developing online scheduling schemes. Indeed, a problem that is beyond the scope of this study is to consider practical online scheduling schemes that achieve

the capacity predicted by this study or approximate it well. An optimal scheme requires joint scheduling of sectors as well as links within each sector to best mitigate co-channel interference. In general, the joint scheduling that is studied here is fairly complex and cannot be achieved in real-time. Hence, one may consider simpler online scheduling schemes and compare their performance to the optimal capacity provided by our tools. This is for further study.

The remainder of this paper is organized as follows. Section II provides background on the study. In Section III, we describe our model, formulate our problem, and present our solution technique. Section IV presents our analytical results and numerical results and discuss the impact of the critical system parameters. Finally, Section V concludes this work.

## II. RELATED WORK

The use of antenna directivity is a well known approach for the control of self-induced co-channel interference within a wireless system. Such approaches have found application in fixed multipoint communications systems where the objective was to maximize frequency reuse and capacity and limit interference within cellular systems, principally by the control of radiated emissions and the judicious assignment of spectrum [10], [11].

A more sophisticated approach to control co-channel interference can be achieved by the use of scheduling and radio-resource coordination. [3] provides an analysis of a distributed scheduling algorithm operating within a fixed system of directive terminals arranged as a hexagonal cell. The prime focus of the work is to gauge the efficacy of a distributed (heuristic) scheduling algorithm that avoids major sources of interference and allows concurrent packet transmission provided Signal Interference Ratio (SIR) conditions are met. A similar study [1] also considers a hexagonal cellular structure but shows that packet error rate and delay can be reduced by coordinating transmission amongst collinear, same frequency sectors located in adjacent cells. The importance of coordinating interfering signals is further shown in [7] where the operation of a single 10-sectored cell is simulated. The work indicates that high throughput is achievable with a heuristic centralized scheduling scheme if the base station has interference information from the remote units. The simulations showed that 3-4 concurrent transmissions could be supported per indoor cell but that the capacity of the proposed technique was sensitive to deployment, especially to movement of objects within the coverage area.

Common to the above studies is the idea that some form of systemic interference knowledge is available in support of scheduling. Such knowledge is then used to control the SIR of co-channel, concurrently transmitted wireless links. Intuitively the antenna radiation characteristics affect performance since both beamwidth overlap and sidelobe power level impact on the SIR performance of sectored systems, especially when adjacent same-frequency sectors overlap within the cell. However, in the above referenced works little consideration is given to the antenna performance or its effects as a component to a sectored cellular system. In our work this becomes a major

consideration and we spend considerable effort examining the effect of radiated interference on capacity. We also examine the effect of user placement. Moreover while most of the cited work was based on heuristics and simulation, we formulate a detailed optimization problem to solve for the optimal scheduling and solve it numerically.

## III. MODEL DESCRIPTION, PROBLEM FORMULATION, AND COMPUTATIONAL TOOLS

### A. Network Model

We consider a single-cell single-hop wireless system. We model the network as a set  $\mathcal{N}$  of *nodes* (or *users*), a gateway  $g$  (note that we use the term base station and gateway interchangeably), and a set  $\mathcal{L}$  of *directed links* between any node  $n \in \mathcal{N}$  and the gateway  $g$ . Let  $|\mathcal{N}| = N$  and  $|\mathcal{L}| = L$ . Each node  $i \in \mathcal{N}$  has a location  $(x_i, y_i)$ . We denote by  $o(l)$  and  $d(l)$  the origin and the destination nodes of  $l$ , respectively. Under our single-cell model,  $L = 2N$  if both uplink and downlink traffics are considered together; otherwise  $L = N$ . In all these cases, we have either  $o(l) = g$  or  $d(l) = g$  for any link  $l$ .

We denote the set of flows by  $\mathcal{F}$  and a flow  $f$  is characterized by its source and its destination. We will consider the downlink case in this paper in which all flows originate at  $g$ . Note that the uplink case will yield similar results if each node uses power control to keep the received power at  $g$  the same. We denote by  $\mathbf{x} = [x_1, \dots, x_L]$  the link load vector, where  $x_l$  is the traffic load on link  $l$ . Since a flow is carried on exactly one link by definition of our flow pattern and because our system is single hop, the throughput of a flow is the same as the traffic carried on the corresponding physical link.

### B. Link Model

We assume one (transmit) power  $P$  and one modulation scheme (or rate). Without loss of generality, we assume that the link rate is  $c_l = 1, \forall l \in \mathcal{L}$ . Our model can be extended to multiple power levels and/or multiple rates. We assume that the base station is equipped with directional antennas and the nodes with omni-directional antennas.

In the absence of interference, we say that a link  $l$  is feasible if the *signal to noise ratio* (SNR) at the receiver (i.e.,  $d(l)$ ) meets the following condition:

$$\text{SNR}_l = \frac{A_g(\theta_{g,d(l)})G_l P}{N_0} \geq \beta \quad (1)$$

where  $G_l$  denotes the channel gain on link  $l$ ,  $A_g(\theta_{g,y})$  denotes the directional antenna gain due to the antenna at node  $g$  in the direction  $\theta_{g,y}$  of node  $y$  taken from the center of the sector created by the antenna of  $g$  (see Figure 1),  $N_0$  is the average thermal noise power in the operating frequency band, and  $\beta$  is the SNR threshold associated with the modulation/coding scheme. The channel gain between two points separated by distance  $d$  is assumed to be given by  $F_l(d/d_0)^{-\eta}$ , where  $d_0$  is the close-in reference distance,  $F_l$  is the shadowing and fading gain and  $\eta$  is the path loss exponent. For this study, we assume  $P$  is high enough to make all the links between the nodes and the gateway feasible in the absence of interference. There is an implicit assumption here that the channel gains are quasi

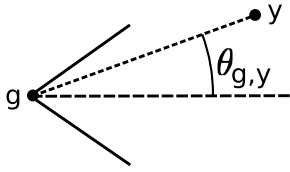


Fig. 1. The relative angles  $\theta_{g,y}$ .

time invariant (when they change, new computations have to be performed).

We assume that the cell is partitioned into  $M$  sectors of identical angles, with each one covered by the same type of directional antenna. A directional antenna is modeled by its *radiation pattern*: a mapping from the angle (relative to the center of a sector) to the antenna gain (see Section III-C). For ease of notation, we replace the case of the gateway  $g$  with multiple antennas by the case of  $M$  co-located gateways  $g_m$  each with a single directional antenna.

Given an antenna model (see Section III-C) and a number of sectors, we now present the link interference model we consider. It extends the *physical interference model* of [4]. We assume that the interference at a given link (at the corresponding receiver) is the cumulative interference from all the links that are active at the same time [6]. In the following, to formulate our joint scheduling problem, we use the concept of an *independent set* (ISet): a set of links that can all operate at the same time without creating any harmful interference<sup>1</sup> at any receiver in the set [5]. We denote by  $\mathcal{I}$  the set of all ISets and by  $\mathcal{I}_l$  the set of ISets that contain link  $l$ . First note that two links  $l = (i, j)$  and  $l' = (i', j')$  cannot be scheduled at the same time if they are in the same sector  $m$ , as they share one node (i.e., the gateway  $g_m$ ) and hence they cannot belong to the same ISet. Denote by  $\delta_{lm}$  the indicator that link  $l$  belongs to sector  $m$ , a set  $s \subseteq \mathcal{L}$  is an ISet only if:

$$\sum_{l \in s} \delta_{lm} \leq 1 \quad \forall m \in \{1, \dots, M\} \quad (2)$$

We now need to take cross-sector interference into account. A set  $s \subseteq \mathcal{L}$  is an ISet iff it meets condition (2) and the following condition:  $\gamma_l \geq \beta \quad \forall l \in s$ , where

$$\gamma_l = \frac{A_{o(l)}(\theta_{o(l),d(l)})G_l P}{N_0 + \sum_{l' \in s: l' \neq l} A_{o(l')}(\theta_{o(l'),d(l)})G_{l'} P} \quad (3)$$

where  $\gamma_l$  is the *signal to interference plus noise ratio* (SINR) of link  $l$  and  $G_{l'}$  is the channel gain from  $o(l')$  to  $d(l)$ .

### C. Antenna Model

We now describe the antenna model for the base station. It is given by:

$$f(\theta; M) = C_1 + (1 - C_1)|2J_1(\alpha(\theta))/\alpha(\theta)|^2,$$

where  $\alpha(\theta) = 4\pi C_{2,M} \sin(\theta/2)$ ,  $J_1(\cdot)$  is the first order Bessel function of the first kind and  $C_1$  and  $C_2$  are parameters that control the sidelobe and beamwidth respectively and are in

<sup>1</sup>We define harmful interference as yielding a SINR lower than  $\beta$ .

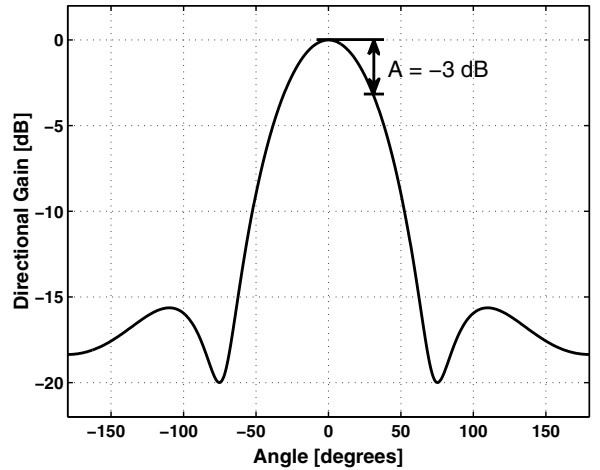


Fig. 2.  $f(\theta; M)$  for  $M = 6$ ,  $C_1 = -20$ dB and  $A = -3$ dB.

general functions of  $M$ , the number of sectors. Note that this model was derived as the average directional gain of a random distributed array in the far-field region [9] and is used in this study for the ease of controlling the beamwidth and residual sidelobe levels. Qualitatively, we expect there to be two sources of impairment to the performance of directional antennas in our system. First, there is the interference caused by the main lobe to sectors adjacent to the sector in which it is centered. Second, there is the interference caused by the sidelobes. Specifically,  $C_1$  directly controls the residual sidelobe level, while by adjusting  $C_{2,M}$ , one can control the amount of attenuation at  $\theta = \pi/M$ , i.e., at the edge of a sector illuminated by the directional antenna. We denote this attenuation by  $A$  (see Figure 2)

*Normalization Issues:* One of the objectives of this study is to analyze the effect of the number of directional antennas at the base station on system capacity. It then becomes important to properly account for transmission power at the base station when the number of antennas varies. In this study, we assume that the total power budget is fixed to  $P_{tot}$ , and that each antenna transmits with identical power  $P_{tot}/M$ .

### D. Problem Formulation

Having defined the ISets earlier, a *link schedule* is an  $|\mathcal{I}|$ -dimensional vector  $\alpha = [\alpha_s]_{s \in \mathcal{I}}$  such that  $\alpha_s > 0$  if  $s \in \mathcal{I}$  is scheduled, otherwise  $\alpha_s = 0$ . We interpret  $\alpha_s$  as the fraction of time allocated to an ISet  $s$ . The reason that we only schedule ISets is straightforward: other link sets incur conflicts and it is hence a waste of resources to schedule them.

We define the capacity of our single cell system comprising  $N$  nodes and  $M$  sectors each with a gateway  $g_m$  as the total throughput offered to all flows when the objective is to maximize the minimum rate offered to any flow, i.e.,  $N$  times the max-min rate. Note that there are several other ways to define ‘‘capacity’’, for example one based on proportional fairness for which capacity is defined with respect to the maximization of the sum of the log of the individual rates, or one based on the maximum sum rate. The problem formulation

and solution technique can be easily extended to any linear objective function such as a weighted sum of the rates or any convex piecewise linear function of the rates.

Our benchmark is a single cell where the gateway is equipped with one omni-directional antenna. In the downlink case with single power and single modulation, each node would then receive a normalized max-min throughput of  $1/N$ . In the case of  $M$  sectors, the max-min throughput is between  $1/N$  and  $M/N$ .

We define the link-set incidence matrix  $Q$

$$q_{l,s} = \begin{cases} 1 & \text{if } l \in s \in \mathcal{I} \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Note that each column  $\mathbf{q}_s$  of  $Q$  is a vector that represents an ISet  $s$ .

Given the network model and the definitions, we formulate our problem in (5)–(8) as maximizing the minimum load among all links.

$$\max_{\mathbf{x}, \boldsymbol{\alpha}} \quad x \quad (5)$$

$$(\mu_{i,j}) \quad \sum_{l \in \mathcal{L}(i,j)} x_l \geq x \quad \forall j \in \mathcal{N} \quad (6)$$

$$(\nu_l) \quad c_l \sum_{s \in \mathcal{I}} q_{l,s} \alpha_s \geq x_l \quad \forall l \in \mathcal{L} \quad (7)$$

$$(\zeta) \quad \sum_{s \in \mathcal{I}} \alpha_s \leq 1 \quad (8)$$

$$\mathbf{x}, \boldsymbol{\alpha} \geq \mathbf{0}$$

The maximization is explicitly taken with respect to the link load allocation  $\mathbf{x}$  and the link scheduling vector  $\boldsymbol{\alpha}$ . We have put the Lagrangian multipliers corresponding to each constraint in parenthesis. In this formulation, we have  $\mu_{i,j} \geq 0$ ,  $\nu_l \geq 0$ , and  $\zeta \leq 0$ .

**Note:** The problem, as formulated in (5)–(8), is a straightforward linear program (LP). However, solving this LP is not trivial. We face two main difficulties: (i) We have to enumerate all the ISets, whose number is extremely large in order to construct the problem. (ii) Even if we are able to enumerate all the ISets, solving an LP with a huge number of variables (i.e.,  $\alpha_s$ ) might not be possible. Hence, to obtain the capacity of a system characterized by  $N$  and  $M$  (as well as other parameters) when  $N$  and  $M$  are large will require developing efficient computational tools.

### E. Solution Technique

As we mentioned in the previous section, directly solving the link scheduling problem (5)–(8) is practically infeasible, due to the huge number of variables  $\alpha_s$  involved for medium to large size systems. However, the number of constraints that involve these variables, i.e., (7) and (8), is not large: there are only  $L + 1$  in total, where  $L = |\mathcal{L}|$  is the number of links. Therefore, the number of non-zero  $\alpha_s$  in an optimal solution is at most  $L + 1$ , according to the Carathéodory's theorem. Then the question becomes: how could we identify those useful ISets (corresponding to those non-zero  $\alpha_s$ ) without enumerating all ISets?

Our solution technique is based on the so called *column generation* technique. Initially, we construct a restricted case of the link scheduling problem, by involving only obvious ISets (in particular, ISets of size 1 or, in other words, links). Such a problem is called *Restrict Master Problem* (RMP). Solving this LP problem gives the primal variable  $\mathbf{x}$  and  $\boldsymbol{\alpha}$ , as well as the dual variables  $\boldsymbol{\mu}$ ,  $\boldsymbol{\nu}$ , and  $\zeta$ . In general,  $\mathbf{x}$  and  $\boldsymbol{\alpha}$  (which are the optimal solutions of the RMP) are not optimal for the original problem, unless certain conditions are met. In particular, if for every  $s \in \mathcal{I}$ , we have

$$\zeta + \sum_{l \in s} c_l \nu_l \leq 0$$

the primal variables are optimal. Such conditions are called *pricing* conditions where the LHS defines the price of  $s$ . The name “pricing” comes from the interpretation of the dual variables in an economic context. If the condition is not met for certain  $s \in \mathcal{I}$ , the current primal variables are not optimal yet. In addition, it also tells us that adding this ISet  $s$  into the RMP may potentially lead to a larger objective value  $x$ . In summary, the solution includes the following steps:

- 1) Initialize the RMP with simple ISets.
- 2) Solve the RMP to obtain both primal and dual variables.
- 3) Verify the pricing condition against eligible ISets. If it is met, terminate the algorithm. Otherwise add the violating ISet to the RMP and go back to 2.

Note that the set of eligible ISets is far smaller than the whole set. The reason is twofold: (1) an ISet that is not maximal<sup>2</sup> is not eligible, as the maximal ISet that contains this ISet is at least as qualified as it is, and (2) an ISet that contains links with zero price<sup>3</sup> is not eligible, as the price of this ISet remains the same if the zero-price links are removed. Moreover, the verification in each iteration stops whenever a violating ISet is found, so it can be made very fast by using techniques such as greedy searching. As a result, this solution technique can dramatically speed up the computation. Furthermore, by stopping the computation before the algorithm terminates, we can obtain a lower bound to the optimal solution.

## IV. ANALYTICAL AND NUMERICAL RESULTS

In this section, we derive some analytical results under simplifying assumptions and compute, using the tool that we have developed, the system capacity for several test cases. We first consider the case where the number of nodes per sector is fixed for a given  $M$  and the nodes are regularly placed at a fixed distance of the base station, i.e., the angle between 2 adjacent nodes is fixed. This we call the deterministic uniform node placement. Under this node placement and assuming that the number of nodes is large and the system is not noise-limited, we derived a formula giving the capacity of the system. Note that, if we call  $\Omega(M)$  the capacity of the system with  $M$  sectors, we have  $1 \leq \Omega(M) \leq M$ . We validate our formulas against the solution of the optimization problem

<sup>2</sup>An ISet that is maximal is not a subset of any other ISets

<sup>3</sup>The price of a link  $l$  is defined as  $c_l \nu_l$ .

given in (5)–(8) obtained by our computational tool based on column generation.

We then relax the assumption that the nodes are deterministically placed and consider two random placements. The first we call “sector random” corresponds to the case where each sector has an equal number of nodes, the nodes being uniformly distributed in the sector. The second we call “disc random” corresponds to the case where the nodes are uniformly distributed over the entire disc. The results show the importance of the placement of the nodes and show that the deterministic uniform node placement assumption can be too “optimistic”.

#### A. Deterministic Uniform Node Placement

In this section, we assume that for a given  $M$ , the number of nodes is the same per sector and the nodes are equally spaced on a circle around the base station.

1) *Analytical Formula:* To derive an analytical formula, we take the noise power to be  $N_0 = 0$  which is equivalent to assuming that co-channel interference is the dominant impairment compared to noise. Nevertheless, even if this is not the case, taking  $N_0 = 0$  always provides an upper bound on performance. While it is difficult to analyze the performance when the number of sectors is small, one can compute the asymptotic throughput in the limit when the number of sectors is large. Our main result in this section is an expression for the limiting value of  $\Omega(M)$  when the number of sectors  $M$  is large.

Specifically, since the antenna gain in any model is normalized so that the maximum gain is one, and the smallest amount of interference is given by  $C_1$ , the largest ISet size is

$$1 + \left\lfloor \frac{1}{\beta C_1} \right\rfloor. \quad (9)$$

Now, for a node at angle  $\theta = \gamma \frac{\pi}{M}$  inside its sector, where  $-1 \leq \gamma \leq 1$ , the antenna gain is

$$g(\gamma; M) = f(\theta; M) = C_1 + (1 - C_1) |2J_1(\alpha(\gamma))/\alpha(\gamma)|^2, \quad (10)$$

where  $\alpha(\gamma) = 4\pi C_{2,M} \sin(\gamma\pi/2M)$ . In the limit when  $M$  is large,  $\alpha(\gamma) \rightarrow \gamma \cdot 2\pi C_{2,M}/M$ , where  $C_{2,M}$  is chosen such that  $g(1; M) = A$  (recall  $A$  is the attenuation of the beam pattern from its peak at the edge of the sector). This implies that  $C_{2,M}/M$  is a constant in the limit of large  $M$ , i.e., in the limit, the antenna gain is given by

$$g(\gamma; M) = C_1 + (1 - C_1) |2J_1(K\gamma)/(K\gamma)|^2, \quad (11)$$

where  $K = 2\pi C_{2,M}/M$  is a constant chosen such that  $g(1; M) = A$ . Since  $A$  is constant, we see that in the limit of large  $M$ , the antenna gain for a node at relative angle  $\gamma$  depends in fact only on  $\gamma$  and not  $M$ . Thus, we denote this limit by  $g(\gamma)$ .

In this limit, for a node at relative angle  $\gamma$ , the maximum size of any ISet to which it belongs is

$$R(\gamma) := 1 + \left\lfloor \frac{g(\gamma)}{\beta C_1} \right\rfloor. \quad (12)$$

Clearly, if each node was in its maximal ISet and only such maximal ISets are scheduled, the system would achieve optimal throughput. For this to be the case, for each node in an ISet of size  $R(\gamma)$ , we must find  $R(\gamma) - 1$  other nodes. However, because  $M$  is large (much larger than  $R(0) = 1 + \lfloor 1/\beta C_1 \rfloor$ ), we can schedule  $R(\gamma)$  sectors simultaneously, all separated enough such that the interference they create to each other is well approximated by  $C_1$ . Thus, if we subdivide each sector into fine slices of width  $\Delta\gamma$ , centered at  $\gamma_1, \gamma_2, \dots$  relative to each sector’s center, then under the strong uniform distribution assumption and a large number of nodes, we can schedule the nodes in slice  $\gamma_i$  in one sector with the nodes in  $R(\gamma_i) - 1$  other sectors, also in the corresponding slices centered at  $\gamma_i$ . Thus, we have a schedule such that each node only operates in its maximal ISets.

Now, let  $t_i$  be the relative amount of time that is spent serving all nodes in all sectors in slices centered at  $\gamma_i$ . Then since the number of nodes in each slice is identical by the strong uniform assumption,

$$t_i = \frac{\Delta\gamma/R(\gamma_i)}{\sum_j \Delta\gamma/R(\gamma_j)}. \quad (13)$$

Thus, the aggregate throughput delivered to all slices centered at  $\gamma_i$  is

$$t_i \times R(\gamma_i) = \frac{\Delta\gamma}{\sum_j \Delta\gamma/R(\gamma_j)}, \quad (14)$$

and the total throughput of the system is

$$C = \sum_i t_i \times R(\gamma_i) \quad (15)$$

$$= \frac{\sum_i \Delta\gamma}{\sum_j \Delta\gamma/R(\gamma_j)}. \quad (16)$$

If we now pass to the limit  $\Delta\gamma \rightarrow 0$ , then the above sums become integrals and

$$C = \frac{\int_{-1}^1 d\gamma}{\int_{-1}^1 1/R(\gamma) d\gamma} \quad (17)$$

$$= \frac{2}{\int_{-1}^1 1 + \lfloor \frac{g(\gamma)}{\beta C_1} \rfloor d\gamma} \quad (18)$$

$$= \left[ \int_0^1 1 + \left\lfloor \frac{g(\gamma)}{\beta C_1} \right\rfloor d\gamma \right]^{-1}, \quad (19)$$

where we have used the fact that the beam pattern is symmetric, i.e.,  $g(\gamma) = g(-\gamma)$ . Thus, (19) provides us with an expression for the value of the capacity when  $M$  is large, the node density sufficiently large and the system is co-channel interference limited (i.e.,  $N_0$  is negligible).

**Comparison with numerical results:** All the results have been obtained using our computational tool for a circular cell of radius 1km,  $N_0 = -100\text{dBm}$ ,  $d_0 = 10\text{m}$ ,  $\eta = 3$ ,  $P_{max} = 9.5\text{dBm}$  and without fading and shadowing. The number of nodes per sector is chosen to be  $\min(20, \text{floor}(200/M))$  and the nodes are placed as described above.

Figure 3 shows the network capacity as a function of  $M$  for  $\beta = 6.4\text{dB}$  and for  $C_1 = -20$  or  $-15\text{dB}$  and  $A = -3$  or

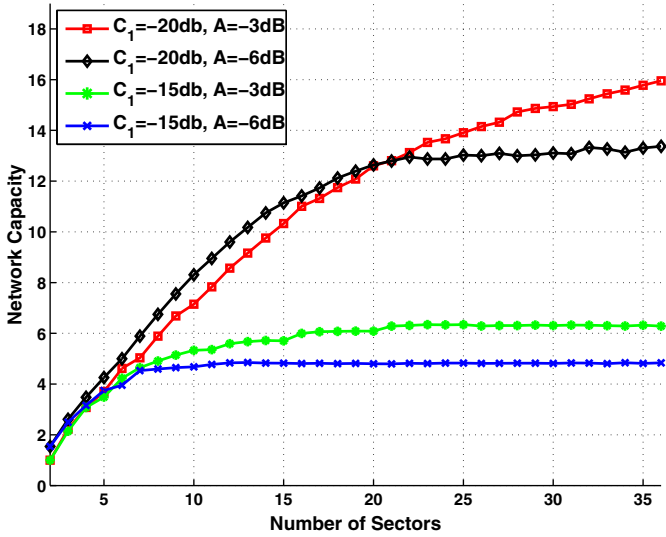


Fig. 3. Network capacity as a function of the number of sectors (deterministic node placement).

$-6\text{dB}$  where  $A$  is the value (in dB) of the gain  $f(\theta; M)$  at  $\theta = \frac{\pi}{M}$  (i.e.,  $C_2(M)$  has to be chosen such that  $A = -3\text{dB}$  (resp.  $-6\text{dB}$ )). Note that in this figure, all points for  $M \leq 5$  are optimal and those for  $M > 5$  are suboptimal. The optimal points are those that our computational tool is able to compute in a reasonable time (i.e., less than 1 hr) while the other ones are obtained by stopping the tool after 1 hr.

The figure clearly illustrates that the capacity increases with  $M$  but that a plateau exists for large  $M$ . From these numerical results, we see that the four plateaus, are (in numerical order) 4.8, 6.2, 13.3 and 15.9. By comparison, the theoretical plateaus from expression (19) are 4.82, 6.17, 14.09 and 18.38. For the cases with  $C_1 = -15\text{dB}$ , the results closely agree. We attribute the differences for  $C_1 = -20\text{dB}$  to be due primarily to the fact that the number of nodes per sector is not sufficiently large to completely take advantage of spatial reuse. These results also demonstrate the importance of the values of  $C_1$  and  $A$ .

To investigate further the impact of  $A$  on the capacity of the system, we have computed the capacity for a system with 3 sectors (resp. 6, 12, 18, 24) for different values of  $A$  with  $C_1 = -20\text{dB}$ . This is shown in Figure 4 for  $\beta = 6.4\text{dB}$ . Interestingly, the figure shows that for a small number of sectors, beyond a minimum value of  $A$ , the performance is relatively insensitive to the value of  $A$  (i.e., the curve is flat) whereas for a large number of sectors, the performance varies greatly with  $A$  (i.e., the curve is bell-shaped). In addition, the optimal value of  $|A|$  (i.e., the value that yields the best capacity) is smaller for a larger number of sectors.

*Definition:* We say that an antenna corresponding to  $M$  sectors has a relatively narrower beam than an antenna corresponding to  $M' \neq M$  sectors if  $A < A'$ .

Hence the figure suggests that for a small number of sectors, relatively narrower beams are better than wider beams while the opposite is true for a large number of sectors.

We explain these results by the fact that in the small number of sectors case, performance is limited by interference between

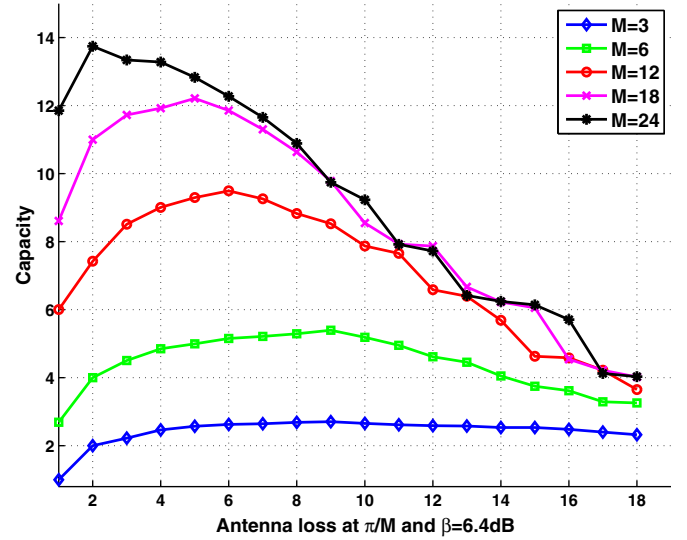


Fig. 4. Network capacity for different number of sectors as a function of the antenna attenuation in dB at the edge of a sector (deterministic node placement).

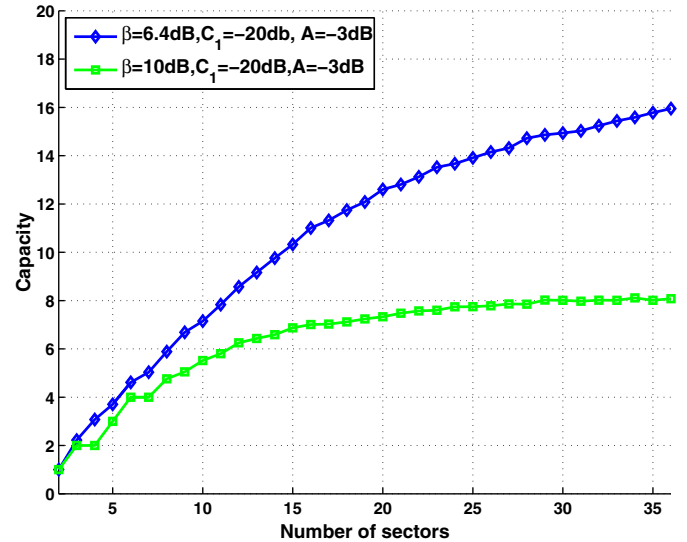


Fig. 5. Network capacity as a function of number of sectors. Impact of  $\beta$

sectors while in the other case, the interference between sectors can be alleviated by sufficiently spacing sectors that are scheduled at the same time, i.e., in the large number of sector case the performance is limited by the attenuation seen by users near the edge of a sector and thus favors a smaller value of  $A$ .

To study the impact of  $\beta$ , we show Figure 5 where the network capacity is represented as a function of  $M$  for  $C_1 = -20\text{dB}$  and  $A = -3\text{dB}$  for  $\beta = 6.4\text{dB}$  and  $\beta = 10\text{dB}$ . Clearly  $\beta$  has a major impact on the network capacity, especially when the number of sectors is large. This is coherent with our analytical study. Note that in this figure, all points for  $M \leq 5$  are optimal and those for  $M > 5$  are suboptimal.

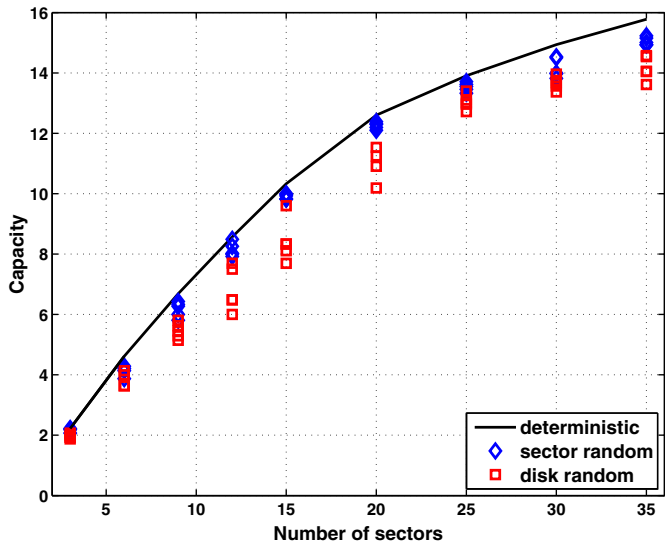


Fig. 6. Network as a function of number of sectors for  $(C_1, A, \beta) = (-20, -3, 6.4)$ dB: comparison between deterministic and random placements

### B. Random Uniform Node Placement

In this section, we assume that for a given  $M$ , the nodes are uniformly distributed in the cell and we consider 2 cases, in the first case (called sector random), the number of nodes is the same in each sector while in the second case (called disk random) we relax this assumption. The number of nodes per sector is chosen to be  $\min(20, \text{floor}(200/M))$ . We ran 5 realizations per value of  $M$  for each of the 2 cases. Fig. 6 shows the network capacity as a function of  $M$  for each realization for the two cases when  $C_1 = -20$ ,  $A = -3$ , and  $\beta = 6.4$ dB. We also show in the figure the “deterministic uniform placement” case. It is clear from the figure that while the deterministic uniform placement qualitatively agrees with the random placement, the random placement shows variability in the achieved capacity and is usually below the deterministic uniform placement which is optimistic. This was to be expected but our tool allowed us to quantify the loss in performance due to “bad” placement. This loss can be as high as 50%. We believe that part of the reason for this is linked to the max-min objective function that we have chosen.

## V. CONCLUSIONS

In summary, we make the following conclusions.

- SDMA can yield excellent capacity for a reasonable number of directional antennas at the base station. The capacity is very dependent on the characteristics of the antenna and hence careful antenna selection is necessary. Indeed, increasing the number of sectors eventually has diminishing returns. In particular, we have found the existence of a plateau in the capacity of the systems. This plateau is mostly a function of the shape of the main lobe in a sector as well as the side lobe interference created to sectors far away. We have found an expression for this plateau under the assumptions of a deterministic uniform node placement and a large number of nodes.

Intuitively, the plateau is due to the fact that for each node in a sector, one can compute a bound on the maximal ISet to which it belongs which is purely a function of the antenna gain at the node’s position, the sidelobe interference and the SINR threshold  $\beta$ . This last point highlights the importance of the antenna model as well as the threshold  $\beta$ . If the number of antennas is moderate so that the plateau has not been reached, then the entire antenna pattern is of interest.

- The distribution of the nodes in the cell and their positions within a sector can also have a large impact on capacity. This shows that in order to obtain the maximum achievable capacity, careful (and cumbersome) scheduling has to be performed. Hence, the network operator is left with two choices. He can either use a simple “sector scheduler” that will work well under any node positions at the cost of a potential sub-optimal capacity or use a more complex scheduler based on node positioning (assuming the positions or channel gains are known) and obtain optimal capacity. This will be studied further in a subsequent paper.
- Another parameter that has an important impact on capacity is the SINR threshold  $\beta$ , hence the choice of modulation is key.
- The calculation of the ISets is prohibitive even when the numbers of users and sectors are moderate. Thus, we have developed a computational tool that computes exact solution whenever possible and otherwise provides a good approximation to the capacity and also provides a schedule. This tool agrees closely with the analytical plateau that we derived and will give us the means to benchmark any sub-optimal scheduling that we will design in the future.
- Our results show that for a small number of sectors, relatively narrower beams (as defined in Section IV) are better compared to wider beams while for a large number of sectors, the opposite is true. This is because, in the small number of sectors case, performance is limited due to interference between sectors while in the other case, the interference between sectors can be alleviated by sufficiently spacing sectors that are scheduled at the same time – the performance then becomes mostly a function of the attenuation at the edge of the sector. Hence the “best” antenna parameters depend on the scenario in which the system is operated.
- The current study has considered a capacity based on max-min throughput, i.e., we are maximizing the throughput given to the worst node. While this is a fair way to allocate system resources, it may not always be appropriate. Specifically, if a node for some reason has a particularly poor channel gain to the base station, a max-min allocation will devote a relatively large fraction of system resources in order to provide rate to a single user. In such a case, other notions such as proportional fair resource sharing may be more appropriate. However note that adapting our tool to non-linear objective functions such as the product of the rates, is not straightforward.
- This study has only considered downlink operation, the

results for uplink are in fact identical under the assumption that nodes control their transmit power such that the received power at the base station is the same for all nodes. This is because in such a case, the interference seen at a base station receiver does not depend on the distance of any user from the base station. Thus the relative strength of the interference to the signal strength in uplink is identical to that in downlink as all other gains (i.e., antenna gains) are the same.

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