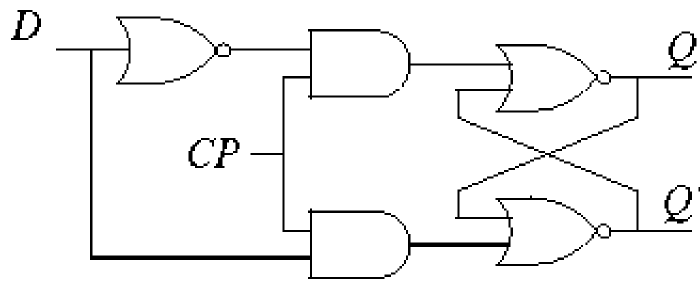
Assignment #6 E & CE 223

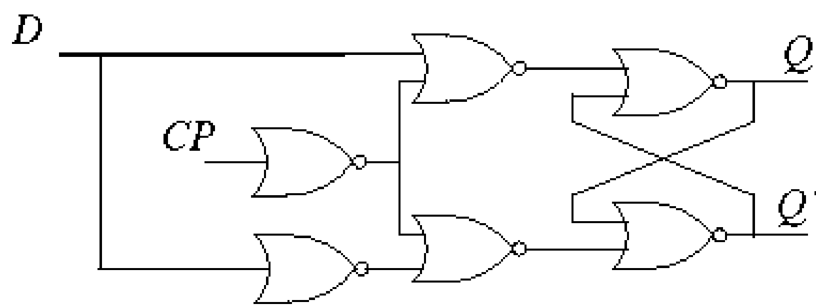
E&CE 223
Assignment 6 - Solutions

1: Mano 6.1 and 6.2

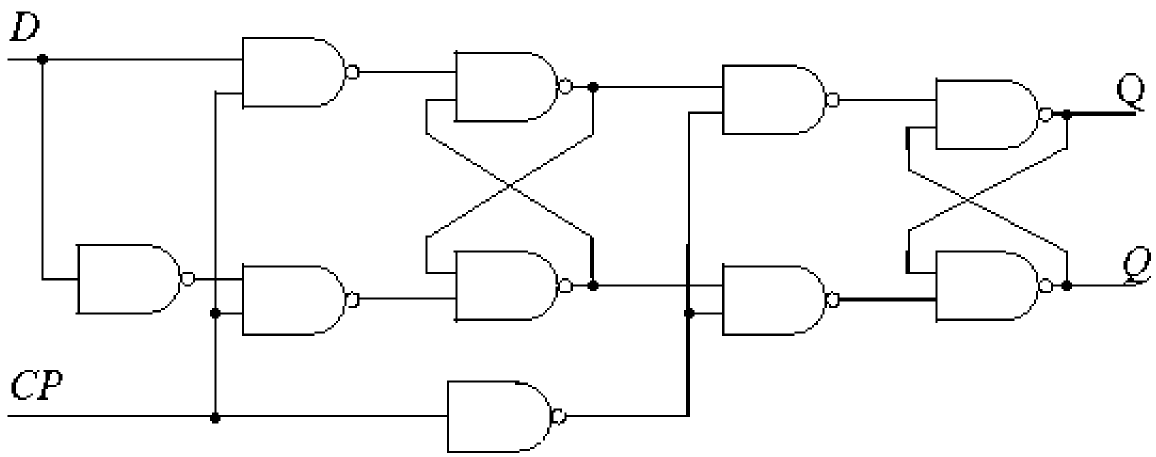
Mano 6.1
See Notes page 187



Mano 6.2



2: Mano 6.4

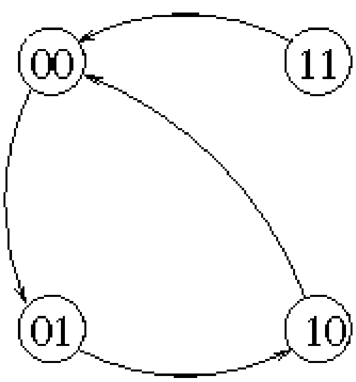


3: Mano 6.9

$T_A = A + B$

$T_B = A' + B$

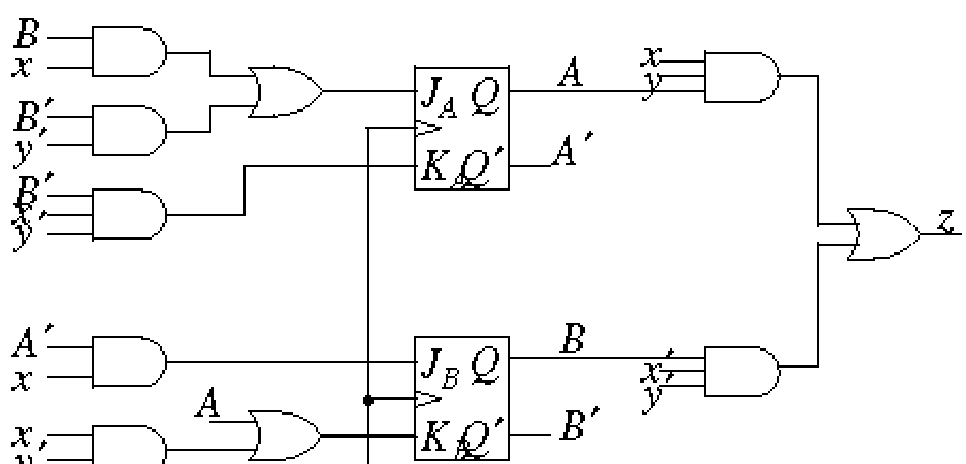
present state AB	T_A	T_B	next state AB
0 0	0	1	0 1
0 1	1	1	1 0
1 0	1	0	0 0
1 1	1	1	0 0



Counter with the sequence 00, 01, 10, 00 etc.

4: Mano 6.12

(a)

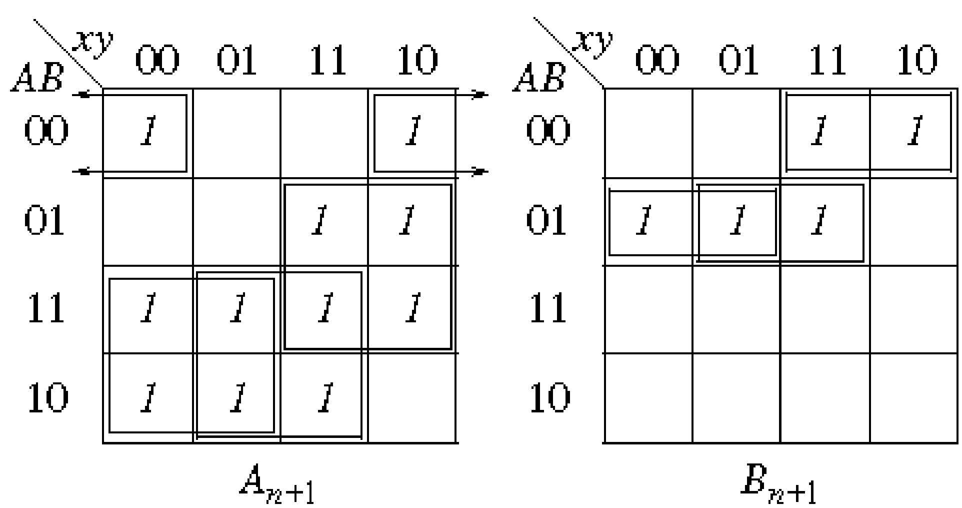


(Mealy machine)

(b)

present state		inputs		FF inputs				next state		output
A	B	x	y	J_A	K_A	J_B	K_B	A	B	z
0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	1	1	1	1	1	0
0	0	1	1	0	0	1	0	0	1	0
0	1	0	0	0	0	0	0	0	1	1
0	1	0	1	0	0	0	0	0	1	0
0	1	1	0	1	0	1	1	1	0	0
0	1	1	1	1	0	1	0	1	1	0
1	0	0	0	1	0	0	1	1	0	0
1	0	0	1	0	0	0	1	1	0	0
1	0	1	0	1	1	0	1	0	0	0
1	0	1	1	0	0	0	1	1	0	1
1	1	0	0	0	0	0	1	1	0	1
1	1	0	1	0	0	0	1	1	0	0
1	1	1	0	1	0	0	1	1	0	0
1	1	1	1	1	0	0	1	1	0	1

(c)



$$A_{n+1} = Ax' + Bx + Ay + A'B'y'$$

$$B_{n+1} = A'B'x + A'Bx' + A'By$$

5: Mano 6.21

Binary up-down counter with enable input E

present state		inputs		next state		FF transitions	
A	*B*	*E*	*x*	*A*	*B*	*A*	*B*
0	0	0	0	0	0	ϕ	ϕ
0	0	0	1	0	0	ϕ	ϕ
0	0	1	0	1	1	α	α
0	0	1	1	0	1	ϕ	α
0	1	0	0	0	1	ϕ	I
0	1	0	1	0	1	ϕ	I
0	1	1	0	0	0	ϕ	β
0	1	1	1	1	0	α	β
1	0	0	0	1	0	I	ϕ
1	0	0	1	1	0	I	ϕ
1	0	1	0	0	1	β	α
1	0	1	1	1	1	I	α
1	1	0	0	1	1	I	I
1	1	0	1	1	1	I	I
1	1	1	0	1	0	I	β
1	1	1	1	0	0	β	β

The transitions in the above table were derived from the current state and next state using the relations:

Q_n	Q_{n+1}	transition
0	0	ϕ
0	1	α
1	0	β
1	1	I

AB \ Ex	00	01	11	10
00	ϕ	ϕ	ϕ	α
01	ϕ	ϕ	α	ϕ
11	I	I	β	I
10	I	I	I	β

A transitions

AB \ Ex	00	01	11	10
00	ϕ	ϕ	α	α
01	I	I	β	β
11	I	I	β	β
10	ϕ	ϕ	α	α

B transitions

Recall that:

J must include α, may include β, I

K must include β, may include α, ϕ

This leads to groupings shown on maps:

solid - J

dashed - K

$J_A = K_A = (Bx + B'x')E$ $\qquad$ $J_B = K_B = E$

Aside:

From the transition tables it is obvious that JK gives a simpler circuit that SR does. Recall that:

S must include α, may include I

R must include β, may include ϕ

$S_A = (Bx + B'x')EA'$ $\qquad$ $S_B = EB$

$R_A = (Bx + B'x')EA$ $\qquad$ $R_B = EB'$

6: Mano 6.22

present state			next state						output	
			x=0			x=1			x=0	x=1
A	B	C	A	B	C	A	B	C		
0	0	0	0	1	1	1	0	0	0	1
0	0	1	0	0	1	1	0	0	0	1
0	1	0	0	1	0	0	0	0	0	1
0	1	1	0	0	1	0	1	0	0	1
1	0	0	0	1	0	0	1	1	0	0
1	0	1	–	–	–	–	–	–	–	–
1	1	0	–	–	–	–	–	–	–	–
1	1	1	–	–	–	–	–	–	–	–

Karnaugh Maps:

$AB \backslash Cx$	00	01	11	10
00	*0*	*1*	*1*	*0*
01	*0*	*0*	*0*	*0*
11	–	–	–	–
10	*0*	*0*	–	–

A_{n+1}

$AB \backslash Cx$	00	01	11	10
00	*1*	*0*	*0*	*0*
01	*1*	*0*	*1*	*0*
11	–	–	–	–
10	*1*	*1*	–	–

B_{n+1}

$AB \backslash Cx$	00	01	11	10
00	*1*	*0*	*0*	*1*
01	*0*	*0*	*0*	*1*
11	–	–	–	–
10	*0*	*1*	–	–

C_{n+1}

The D flip-flip excitation can be derived directly from the K-map. We will generate the transition maps and do D, SR, JK and T from that common base.

Transition Maps:

AB \ Cx	00	01	11	10
00	φ	α	α	φ
01	φ	φ	φ	φ
11	–	–	–	–
10	β	β	–	–

A

AB \ Cx	00	01	11	10
00	α	φ	φ	φ
01	I	β	I	β
11	–	–	–	–
10	α	α	–	–

B

AB \ Cx	00	01	11	10
00	α	φ	β	I
01	φ	φ	β	I
11	–	–	–	–
10	φ	α	–	–

C

	"Set Inputs"		"Reset Inputs"	
D	$D = (\alpha, I)$	$X = (-)$		
SR	$S = (\alpha)$	$X = (-, I)$	$R = (\beta)$	$X = (-, \phi)$
JK	$J = (\alpha)$	$X = (-, I, \beta)$	$K = (\beta)$	$X = (-, \phi, \alpha)$
T	$T = (\alpha, \beta)$	$X = (-)$		

D flip-flops:

AB \ Cx	00	01	11	10
00	φ	α	α	φ
01	φ	φ	φ	φ
11	–	–	–	–
10	β	β	–	–

A

AB \ Cx	00	01	11	10
00	α	φ	φ	φ
01	I	β	I	β
11	–	–	–	–
10	α	α	–	–

B

AB \ Cx	00	01	11	10
00	α	φ	β	I
01	φ	φ	β	I
11	–	–	–	–
10	φ	α	–	–

C

$D_A = A'B'x \qquad D_B = C'x' + A + BCx \qquad D_C = A'B'x' + Ax + Cx'$

Note these equations could have been developed directly from the K-map.
Self correcting?
Yes. $D_A = 0$ in all invalid states, causing A to be reset to a valid state.

SR flip-flops:

A

AB \ Cx	00	01	11	10
00	φ	α	α	φ
01	φ	φ	φ	φ
11	–	–	–	–
10	β	β	–	–

B

AB \ Cx	00	01	11	10
00	α	φ	φ	φ
01	I	β	I	β
11	–	–	–	–
10	α	α	–	–

C

AB \ Cx	00	01	11	10
00	α	φ	β	I
01	φ	φ	β	I
11	–	–	–	–
10	φ	α	–	–

$S_A = A'B'x \qquad R_A = A$

$S_B = C'x' + A \qquad R_B = A'C'x + A'Cx'$

$S_C = A'B'x' + Ax \qquad R_C = A'x$

Note: R_B could be simpler, but this would cause $R_B S_B$=1 in some invalid states, which might cause the flip-flop to enter a metastable state if these invalid states were entered due to a malfunction. Self correcting?

Yes. R_A=A. If any of the invalid states are entered, A is reset to a valid state.

JK flip-flops:

A

AB \ Cx	00	01	11	10
00	φ	α	α	φ
01	φ	φ	φ	φ
11	–	–	–	–
10	β	β	–	–

B

AB \ Cx	00	01	11	10
00	α	φ	φ	φ
01	I	β	I	β
11	–	–	–	–
10	α	α	–	–

C

AB \ Cx	00	01	11	10
00	α	φ	β	I
01	φ	φ	β	I
11	–	–	–	–
10	φ	α	–	–

$J_A = B'x \qquad K_A = 1$

$J_B = C'x' + A \qquad K_B = C'x + Cx'$

$J_C = A'B'x' + Ax \qquad K_C = x$

Self correcting?
Yes. K_A=1. If any of the invalid states are entered, A is reset or toggled to a valid state.

T flip-flops:

A

AB \ Cx	00	01	11	10
00	φ	α	α	φ
01	φ	φ	φ	φ
11	–	–	–	–
10	β	β	–	–

B

AB \ Cx	00	01	11	10
00	α	φ	φ	φ
01	I	β	I	β
11	–	–	–	–
10	α	α	–	–

C

AB \ Cx	00	01	11	10
00	α	φ	β	I
01	φ	φ	β	I
11	–	–	–	–
10	φ	α	–	–

$T_A = B'x + A \qquad T_B = B'C'x' + A + BCx' \qquad T_C = A'B'C'x' + Ax + Cx$

Self correcting?
Yes. $T_A = \cdots + A$. In all the invalid states this causes A to be toggled, leading to a valid state.

In all cases the output is:

y

AB \ Cx	00	01	11	10
00		1	1	
01		1	1	
11	X	X	X	X
10			X	X

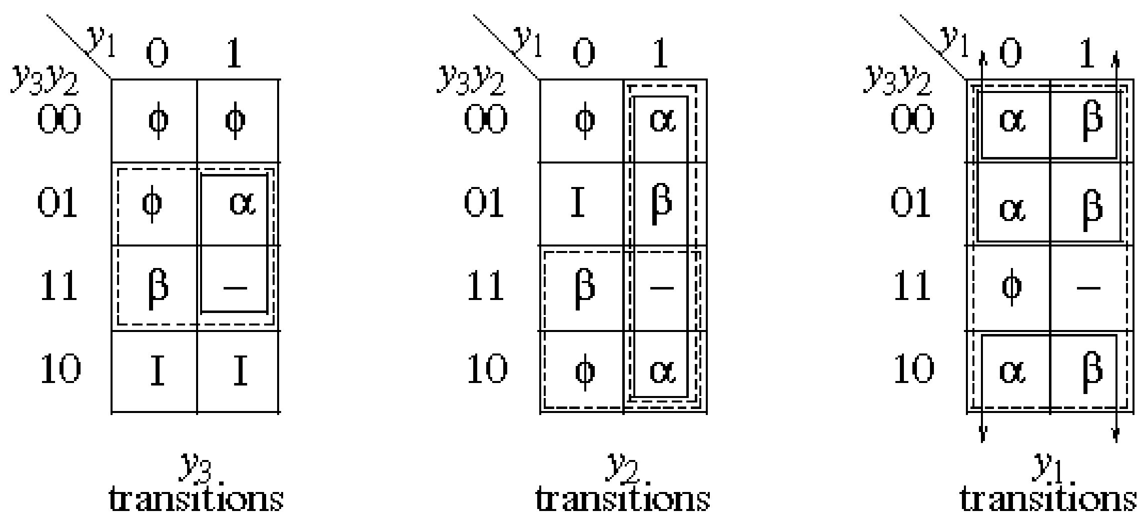
$y = A'x$

7: Mano 6.25 (a) and (b)

(a) JK flip-flops, count sequence 0,1,2,3,4,5,6

present state			next state			F/F transition		
y_3	y_2	y_1	y_3	y_2	y_1	y_3	y_2	y_1
0	0	0	0	0	1	φ	φ	α
0	0	1	0	1	0	φ	α	β
0	1	0	0	1	1	φ	I	α
0	1	1	1	0	0	α	β	β
1	0	0	1	0	1	I	φ	α
1	0	1	1	1	0	I	α	β
1	1	0	0	0	0	β	β	φ
1	1	1	–	–	–	–	–	–

Transition Maps:



$J_3 = y_2 y_1$ $\quad J_2 = y_1$ $\quad J_1 = y_1' + y_2'$

$K_3 = y_2$ $\quad K_2 = y_3 + y_1$ $\quad K_1 = 1$

Check of unused states:

(111) ⇒ (000) OK - no trap

(b) D flip-flops, count sequence 0,1,2,4,6

Do not need transition table - just use k-map of next states

present state y_3	y_2	y_1	next state y_3	y_2	y_1
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	–	–	–
1	0	0	1	1	0
1	0	1	–	–	–
1	1	0	0	0	0
1	1	1	–	–	–

y_3y_2 \ y_1	0	1
00	0	0
01	1	–
11	0	–
10	1	–

$y_3(n+1)$

y_3y_2 \ y_1	0	1
00	0	1
01	0	–
11	0	–
10	1	–

$y_2(n+1)$

y_3y_2 \ y_1	0	1
00	1	0
01	0	–
11	0	–
10	0	–

$y_1(n+1)$

$D_3 = y_3 \oplus y_2$

$D_2 = y_1 + y_3 y_1'$

$D_1 = y_3' y_2' y_1'$

Self correcting:

(011) ⇒ (110) OK, no trap

(101) ⇒ (110) OK, no trap

(111) ⇒ (010) OK, no trap