Sensor-Array Signal Processing

ECE 603: Statistical Signal Processing

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Sensor-Array Signal Processing -- Why and for What?

- A wireless signal, propagating through the atmosphere / the sea / outer space is a spatio-temporal phenomenon.
- Such a wireless signal may be empirically measured
  - over time (e.g., by one immobile sensor over many time-sampling instants),
  - over space (for example, by several spatially separated sensors), and
  - over polarization (for example, by antennas of different polarizations).
- Sensor-array space-time signal processing's typical objectives include:
  - Enhanced reception of the signal-of-interest (SOI), e.g. in wireless communications
  - Rejection of interference, e.g. in multiple-user access communications, CDMA
  - Estimation of the SOI's azimuth-elevation direction-of-arrival (DOA), e.g. in radar/sonar tracking of hostile incoming missiles, FCC's 911 geolocation in the USA.
  - Focusing of a to-be-transmitted signal towards a targeted azimuth-elevation angular sector to minimize interference to other sectors, e.g. in base-station-to-mobile downlink cellular communications.
- Sensor-array space-time signal processing's applications: wireless communications (land-mobile, indoor, satellite, space, underwater, radio-wave, infrared), defense electronics (radar/sonar), air-acoustic microphone-array, biomedical imaging, seismology.

The Space-Time Propagation Model

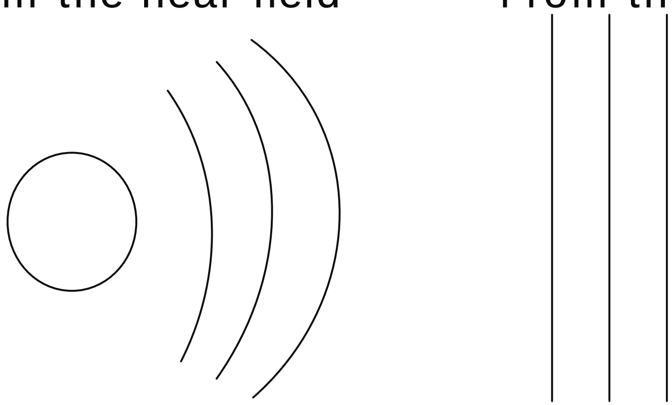
First-Order Assumptions on the Incident Source & the Propagation Channel:

(relaxation of these assumption leads to active research topics)

- The signal's source lies in the far-field; hence, the impinging wave-front is plane-like.
- The signal's source looks spatially point-like to the receiver; hence, the impinging wave-front has a well-defined direction-of-arrival.
- The propagation proceeds along a line-of-sight (LOS) path with no diffraction.

Wavefront originated from the near field

Wavefront originated From the far field



A far-field source may be approximated as a plane-wave.

The Space-Time Propagation Model

$\mathbf{r}$: emitting source's position vector relative to the receiver

$\|\mathbf{r}\|$: emitting source's distance from the receiver

θ_{el}: emitting source's elevation angle from the receiver

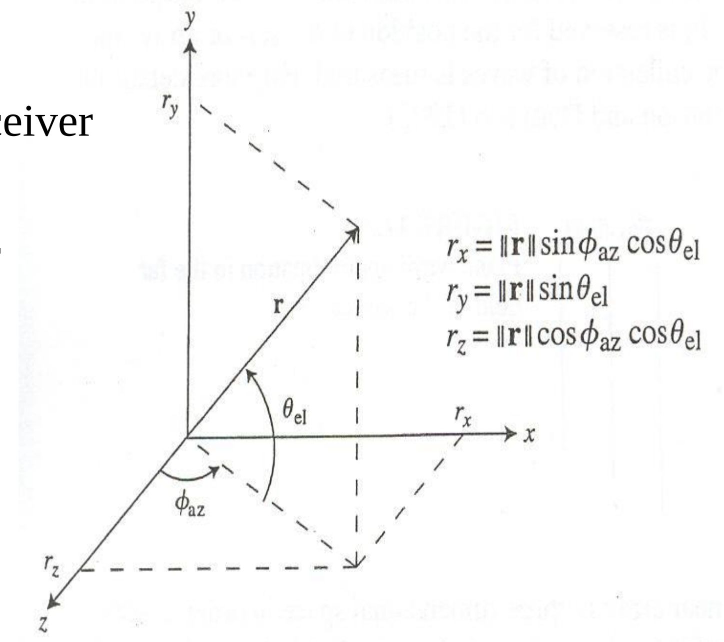
ϕ_{az}: emitting source's azimuth angle from the receiver

A: impinging signal's complex-valued magnitude

c: impinging signal's propagation speed

F_c: impinging signal's carrier frequency

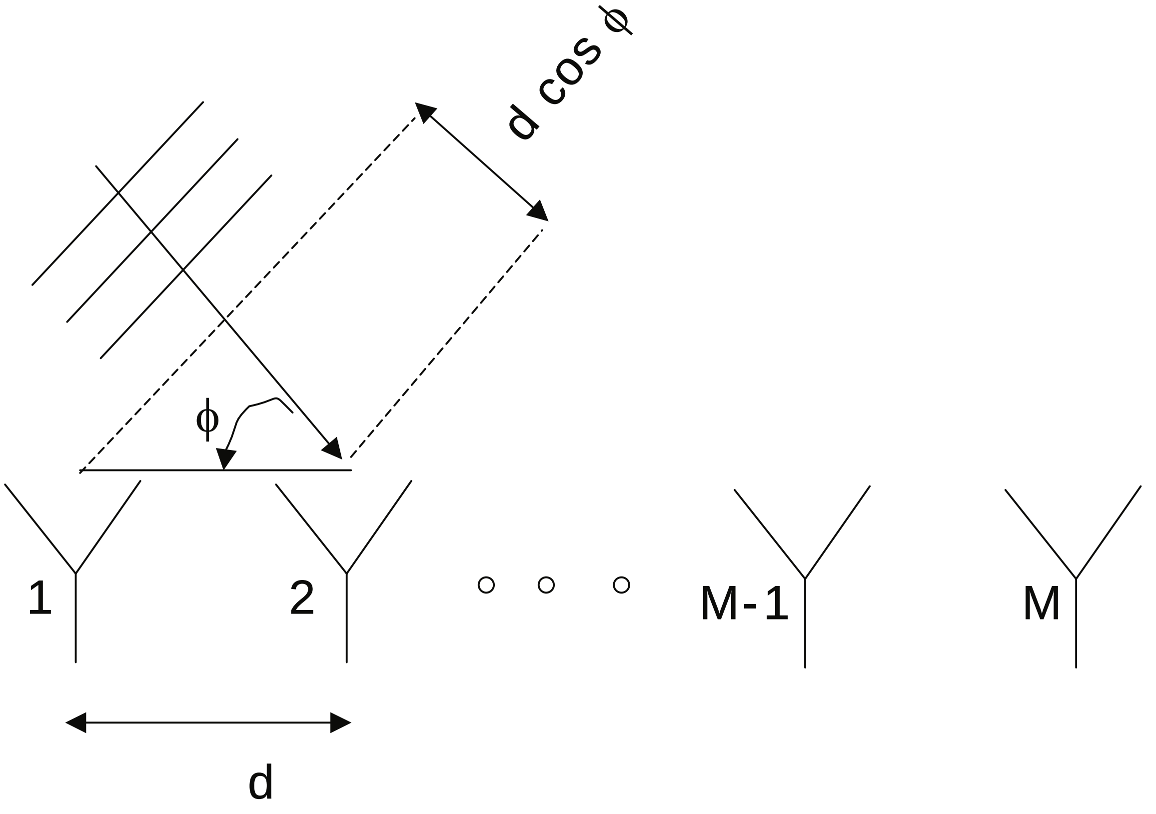
$\lambda = c / F_c$: impinging signal's wavelength



$$s(t, \mathbf{r}) = \frac{A}{\|\mathbf{r}\|} \exp\left(j 2\pi F_c \left(t - \frac{\|\mathbf{r}\|}{c} \right) \right)$$

Uniform Linear Array (ULA)

- ULA: An array of identical omni-directional sensors are lined up on a straight line, with equal inter-sensor spacing (d) between all pairs of adjacent sensors.
- An impinging plane wave-front will arrive at the (m-1)th sensor before arriving at the mth sensor, traveling an additional distance of $d \cos\phi$.
- A distance of ($d \cos\phi$) corresponds to a phase-difference of $2\pi(d/\lambda) \cos\phi$



The baseband information signal: $s_0(t)$

The carrier-modulated and transmited signal propagating through the medium:

$$\tilde{s}_0(t) = s_0(t)\cos\left(2\pi F_c t\right)$$

The data collected by the mth sensor at time t:

$$\tilde{x}_m(t) = h_m(t,\phi) * \tilde{s}_0(t-\tau_m) + \tilde{w}_m(t)$$

$\Updownarrow$ Fourier transform, with the baseband signal's bandwidth << F_c

$$\tilde{X}_m(f) = H_m(f,\phi)\,\tilde{S}_0(f)\exp\left(-j2\pi f\tau_m\right) + \tilde{W}_m(f)$$

where

$h_m(t,\phi)$ is the mth sensor's impulse-response for the arrival-angle ϕ,

$\tilde{w}_m(t)$ is the additive noise at the mth sensor at time t.

$\tilde{S}_0(f) = S_0(f-F_c) + S_0^*(-f-F_c) =$ Fourier transform of $\left\{\tilde{s}_0(t)\right\}$

After frequency down-conversion at the mth sensor,

$$X_m(f) \quad = \quad H_m(f + F_c, \phi)\, S_0(f) \exp\left(-j2\pi f \tau_m\right) + W_m(f)$$

Assuming $H_m(f + F_c, \phi) = H(F_c)$ within the bandwidth of $\tilde{S}_0(f)$ at all ϕ for all sensors,

$$X_m(f) \quad = \quad H(F_c)\, S_0(f) \exp\left(-j2\pi f_c \tau_m\right) + W_m(f)$$

$\Downarrow$ Inverse Fourier transform & time-sampling, the data collected by sensor m at discrete-time n,

$$x_m(n) \quad = \quad H(F_c)\; s_0(n) \exp\left(-j2\pi f_c \tau_m\right) + w_m(n)$$

The data collected by all M sensors at discrete-time n,

$$\mathbf{x}(n) \quad = \quad \left[x_1(n) \quad x_2(n) \quad \cdots \quad x_M(n)\right]^T \quad = \quad \sqrt{M}\; H(F_c)\; \mathbf{v}(\phi)\; s_0(n) + \mathbf{w}(n)$$

where

$$\mathbf{v}(\phi) \quad = \quad \frac{1}{\sqrt{M}} \left[\exp\left(-j2\pi f \tau_1\right) \quad \exp\left(-j2\pi f \tau_2\right) \quad \cdots \quad \exp\left(-j2\pi f \tau_M\right)\right]^T$$

$$= \quad \frac{1}{\sqrt{M}} \left[1 \quad \exp\left(-j2\pi \frac{d\cos\phi}{\lambda}\right) \quad \cdots \quad \exp\left(-j2\pi \frac{(M-1)\, d\cos\phi}{\lambda}\right)\right]^T$$

$$= \quad \frac{1}{\sqrt{M}} \left[1 \quad \exp\left(-j2\pi u(\phi)\right) \quad \cdots \quad \exp\left(-j2\pi (M-1)\, u(\phi)\right)\right]^T$$

is the normalized array manifold of the an ULA of identical omni-directional sensors.

Moreover, $u(\phi) = \dfrac{d\cos\phi}{\lambda}$ represents the incident source's "direction cosine“,

Iff $d \,/\, \lambda \leq 0.5$, $u(\phi)$ is one-to-one mapped to $\exp(-j2\pi u(\phi))$. Otherwise, the sensor-array is “thin” or “sparse”.

First-Order Assumptions on the Uniform Linear Array:

(relaxation of these assumptions leads to active research topics)

- The incident source's bandwidth is "narrowband" in that the signal's bandwidth is much smaller than the inverse of the time needed to travel across the array's length. This "narrowband" assumption implies a time-delay equals a spatial phase-shift. (Note that this "narrowband" assumption differs from another narrowband assumption defined with respect to the propagation channel.)
- Each sensor's gain/phase/polarization response is constant across all arrival directions and the incident signal's bandwidth,
- All sensors have identical gain/phase/polarization response.
- No mutual coupling exists among the sensors.
- The uniform spacing is perfect (I.e., there is no location uncertainty).