

# SUPERCONDUCTING TRANSMISSION LINE PERIODICALLY-LOADED BY A NORMAL TRANSMISSION LINE AND DIELECTRIC GAP

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**Abstract:** This paper presents the dispersion curves of two different superconductive periodic structures. The effect of slow-wave propagation can be easily seen in the superconductor-normal (S-N) periodic transmission line. There is a bandgap in the case of superconductor-insulator (S-I) in dispersion diagram.

## I. INTRODUCTION

Microwave superconductivity is one of the most important applications of superconductors in engineering and technology. The negligible loss in superconductors opens a wide range of applications in realization of low-power, high speed and miniaturized high-frequency components. Sensitivity of superconductivity to temperature, magnetic field and driving current can potentially offer the possibility of realizing tunable microwave devices. Enormous efforts have been done to take benefit from these fundamental characteristics of superconductors in microwave devices and systems such as couplers, filters, multiplexers, front-end receivers, phase shifters [1] and even in realizing metamaterials [2], but the development of new superconducting microwave devices is still under progress.

Superconducting transmission lines are one of the most essential elements in microwave and millimeter-wave integrated circuits. In fact, there are two important properties in a superconductive film which make it plausible in superconducting transmission line: very small amount of loss and associated inductance due to the kinetic energy of paired electrons (superelectrons). Ultra-low-loss, ultra-low dispersion, slow-wave propagation and nonlinearity in the kinetic inductance are the main features of superconducting transmission lines.

One of the most important classes of electromagnetic structures is based on the periodic structures. Periodic structures are conceived to control the propagation of electromagnetic waves, so periodic structures have found many applications in microwave systems such as electromagnetic band gap, frequency selective surfaces, microwave filters, high-impedance surfaces, slow-wave structures and array of antennas [3], [4].

In this paper an attempt has been made to find the features and characteristics of two novel superconducting periodic structures, namely, the periodic superconductor-normal conductor (S-N) and the periodic superconductor-insulator (S-I) transmission lines. For both cases, the dispersion analyses are performed and the results are presented.

## II. PERIODIC S-N TRANSMISSION LINE

*Structure.* The structure under the study consists of a superconductive parallel-plate transmission line which some parts of the superconductor film periodically has been replaced by normal metal. The ground plane is assumed to be superconductor, but it can be swapped by normal conductor. The geometry and parameters are illustrated in Fig.1-a.

*Theory and Analysis.* Consider that electromagnetic wave is propagating along the z-direction. The resistance of superconductor at low frequencies is negligible, but for higher frequency it could even more than normal materials. Hence, to have a rigorous modeling, we assume complex propagation constant for the traveling wave down the transmission line. Now by using distributed circuit model of transmission line and field analysis of the line, distributed circuit

elements for both superconductor and normal part can be obtained [5]. The next step is to employ ABCD matrix or Transfer Matrix formalism to find the relation between input and output currents and voltages for each unit cell. Finally, by assistance of Floquet theorem [6], the dispersion relation for this structure can be found as follows

$$\gamma l = \cosh^{-1} \left\{ \cosh(\gamma_s l_s) \cosh(\gamma_n l_n) + \frac{1}{2} \left( \frac{Z_{0s}}{Z_{0n}} + \frac{Z_{0n}}{Z_{0s}} \right) \sinh(\gamma_s l_s) \sinh(\gamma_n l_n) \right\} \quad (1)$$

where  $\gamma(\omega) = \alpha(\omega) + j\beta(\omega)$  is a complex propagation constant for the periodic structure  $\gamma_s$ ,  $\gamma_n$ ,  $Z_s$  and  $Z_n$  are complex propagation constants and characteristic impedance for superconducting and normal part of each unit cell, respectively. Other parameters are shown in Fig.1-a.

*Numerical Results.* The dispersion curve for various combinations of superconductor and normal part is shown in Fig.1-b and Fig.2-a. As inverse cosine hyperbolic function has a branch cut, therefore care should be taken to have correct results. Fig.1-b clearly shows that how superconductor part slows down the wave propagation due to the extra kinetic inductance present in the line. Fig.2-b demonstrates that attenuation constant increases at higher frequencies.

## II. PERIODIC S-I TRANSMISSION LINE

*Structure.* This structure contains periodic dielectric gaps along the superconductive transmission line which is illustrated in Fig.3-a.

*Theory and Analysis.* As long as the width of the dielectric gap is much less than the wavelength of propagation, this discontinuity can be characterized by an equivalent circuit model; otherwise, this should be considered as a dielectric waveguide and it could be modeled by scattering matrix. There are a number of ways to model this gap [7]; we used the equivalent  $\pi$ -network [8] as shown in Fig.3-b.

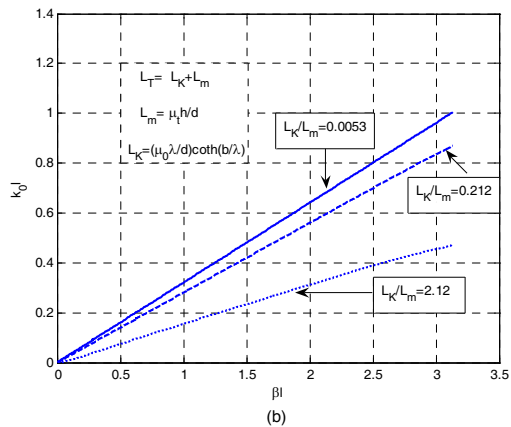
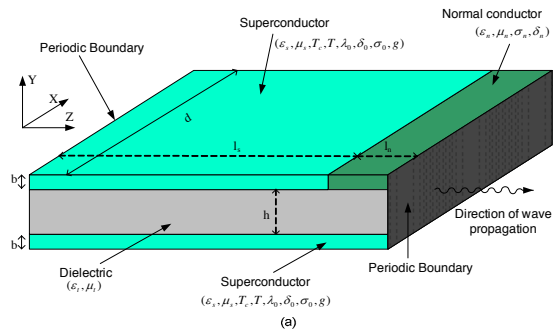
*Numerical Results.* The dispersion curve was depicted in Fig.3-c for the first Brillouin zone. Due to the gap and Bragg condition there is a bandgap in the dispersion diagram. The width of the bandgap can be controlled by changing the width of the gap in structure as shown in Fig.3-c.

## III. CONCLUDING REMARKS

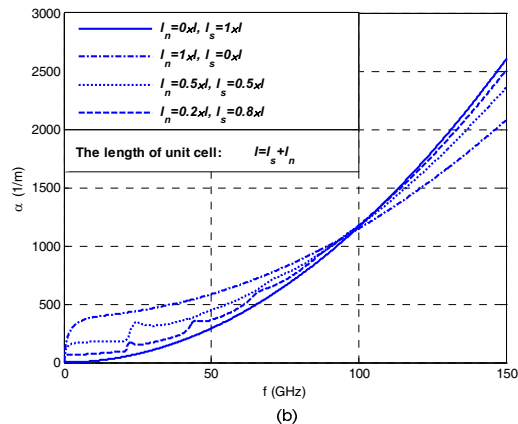
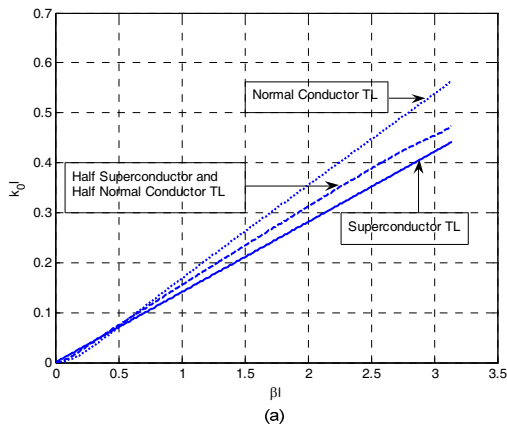
Dispersion engineering has been demonstrated in S-N and S-I transmission lines. This opens up a number of possibilities to design microwave and mm-wave filters, resonators and parametric devices such as amplifiers and mixers. This analysis can be extended into the study of electromagnetic waves in arrays of Josephson Junctions.

## REFERENCES

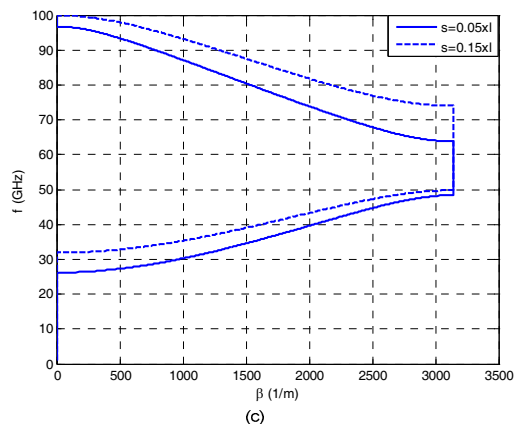
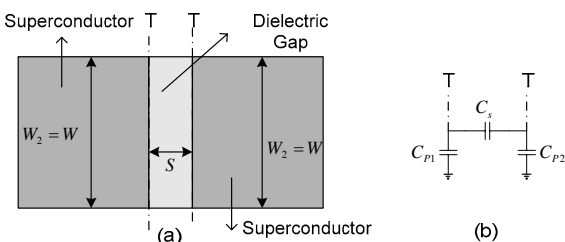
- [1] M. J. Lancaster, "Passive microwave device and applications of high-temperature Superconductors", Cambridge University Press, 1997.
- [2] H. Salehi and A. H. Majedi, "Analysis and design of superconducting left-handed transmission lines", *IEEE Trans. on Appl. Super.*, vol. 15, no. 2, June 2005, pp 996-999.
- [3] N Lizarides, "Propagation of electromagnetic waves in spatially modulated Josephson Junction", *Superconductor Science and Technology*, vol.18, 2005, pp. 73-81.
- [4] H. R. Mohebbi, "Mode Matching Technique for Calculating the Dispersion Curve of One-Dimensional Periodic Structures", Master Thesis, Concordia University, April, 2006.
- [5] T. P. Orlando, K.A. Delin, "Foundations of Applied Superconductivity", Addison-Wesley, 1990.
- [6] R. E. Collin, "Foundations for Microwave Engineering," McGraw-Hill, 1992. [7] K. C. Gupta, R. Garg, I. Bahl and P. Bhartia, "Microstrip Lines and Slotlines", Artech House, 1996.
- [8] M. Kirschning, R. H. Jansen and N.H. Koster, "Accurate model for open end effect of microstrip lines", *Electron Let.*, vol.17, Feb. 1981, pp.123-125.



**Fig.1.** (a) Geometry of the periodic S-N transmission line (b) Dispersion curves for frequencies 0-150 GHz;  $L_k$  is a kinetic inductance due to superconductors;  $L_t$  is a transmission inductance due to the dielectric. YBCO, copper and alumina were used as high temperature superconductor, normal metal and dielectric. For all simulations:  $l=l_n+l_s=1\text{mm}$ ,  $b=100\text{nm}$ ,  $h=1\mu\text{m}$ ,  $d=10\text{h}$ ,  $T=77\text{K}$ ,  $T_c=92\text{K}$ ,  $\lambda_L=250\text{nm}$ ,  $\sigma_0=1.7\times 10^6\text{ S/m}$ ,  $\sigma_n=5.8\times 10^7\text{ S/m}$ ,  $\epsilon_r=9.65\epsilon_0$ ,  $\epsilon_s=\epsilon_n=\epsilon_0$ ,  $\mu_t=\mu_s=\mu_n=\mu_0$ .



**Fig.2.** (a) Dispersion curves for Fig.1-(a) for frequencies from 0-150GHz (b) Attenuation constant for Fig.1-(a)  $l_s$  and  $l_n$  are the lengths of superconductor and normal part in each unit cell. The length of unit cell is  $l=1\text{mm}$ .



**Fig.3.** (a) Geometry of the S-I periodic transmission line from top view,  $h=400\mu\text{m}$ ,  $w_1=w_2=d$  (b) Its equivalent circuit model (c) Dispersion curves for this structure. The width of the gap is denoted by  $s$ .