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Adaptive control of a class of discrete-time affine nonlinear systems[1]

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Abstract

The adaptive control problem is addressed in the paper for a class of discrete-time affine nonlinear input/output stochastic models with linear unknown parameters. The controller is a certainty equivalence weighted one-step-ahead control and is constructed by using the weighted-least-squares and random regularization methods. Global stability of the closed-loop systems is established, which shows that arbitrarily large growth rate is allowed for the multiplicative nonlinear part of the systems. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Consider the following discrete-time nonlinear stochastic system:

$$y_{t+1} = \alpha^\tau f(\varphi_t) + \beta^\tau g(\varphi_t) u_t + w_{t+1}, \tag{1}$$

$$\varphi_t = (y_t \cdots y_{t-p+1}, u_{t-1} \cdots u_{t-q})^\tau \tag{2}$$

where y_t, u_t and w_t are the output, input and random noise sequences, respectively, $\alpha \in \mathbb{R}^m$ and $\beta \in \mathbb{R}^l$ are the unknown parameter vectors, $f(\cdot)$ and $g(\cdot)$ are nonlinear vector functions defined on $\mathbb{R}^{p+q}$.

In terms of the connections with the input u_t in Eq. (1), $f(\cdot)$ may be called additive nonlinearity, while $g(\cdot)$ multiplicative nonlinearity, and the system (1) may be (formally) regarded as an affine nonlinear input/output model. Obviously, it includes several standard models of interest.

During the past several decades, much effort has been devoted to the adaptive control of linear stochastic models where in (1) $f(\varphi_t) = \varphi_t$ and $g(\varphi_t) = 1$. Fairly complete theory is now available for both minimum-phase (cf. e.g. [4] and the references therein) and nonminimum-phase (cf. e.g. [5]) linear stochastic systems, and efficient methods for both design and analysis have been relatively well developed. These methods can be directly applied to a class of nonlinear models where in Eq. (1) the additive nonlinear function f satisfies a linear growth (LG) condition and the multiplicative function g is bounded from both below and above [9]. Naturally, it is desirable to remove or relax the LG condition on f and the boundedness of g. Unfortunately, the relaxation of the LG condition has turned out to be the key technical difficulty in the discrete-time case. This is so even for deterministic systems, since the existing design and analysis methods which are successful in the continuous-time case (cf. e.g. [8]) do not seem to be applicable to the discrete-time case (see e.g. [7] for related discussions).

Recently, it has been shown in [6] that there are in fact some fundamental limitations in relaxing the LG condition in the discrete-time case. To be precise, for nonlinear stochastic models with only additive non-

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linearity (i.e. $g(\cdot)=1$ in Eq. (1)), it has been shown in [6] that in order for the system to be globally stabilizable, the growth rate of $f(x)$ should be slower than $O(||x||^4)$. Further investigation in [10] shows that the growth rate allowed for global stabilization actually depends on the dimension m of the unknown parameter vector α, and when m increases, the restriction on $f(x)$ approaches linear growth.

For the multiplicative nonlinear part, however, the boundedness condition on $g(\cdot)$ may not be necessary, and the situation seems to be quite different. Indeed, for the simplest (but nontrivial) bilinear model where in (1), $f(\varphi_t)=y_t$, and $g(\varphi_t)=[1, y_t]^\tau$, a complete stability result was given in [6], which improves on an earlier result in [3] and shows that $g(\cdot)$ is allowed to have at least linear growth rate.

The main objective of this paper is to present some general conditions under which the system (1) is adaptively stabilizable. We will find that, unlike the additive part, the multiplicative part is allowed to have arbitrarily fast nonlinear growth rate.

2. The main results

We need the following conditions for the system (1):

(A1) There exist constants K_1 and K_2 such that

$$||f(x)|| \leqslant K_1 + K_2||x||, \quad \forall x \in \mathbb{R}^{p+q}.$$

(A2) $p \geqslant q$, and there exists a decomposition $p=p_1+p_2$ with $p_2 \geqslant \max(q, p_1)$, $p_1 \geqslant 0$ such that the function

$$g(x)=g(x_1,x_2,x_3), \quad x_1 \in \mathbb{R}^{p_1},\ x_2 \in \mathbb{R}^{p_2},\ x_3 \in \mathbb{R}^q$$

is uniformly bounded for bounded x_2, and uniformly tends to ∞ as $||x_2|| \to \infty$, where the uniformity is w.r.t. $(x_1,x_3) \in \mathbb{R}^{p_1+q}$.

(A3) There exists a nonzero multivariate polynomial function $P(\gamma)$, $\gamma \in \mathbb{R}^l$, such that the set

$$\mathscr{B} \triangleq \{\gamma\colon P(\gamma) \neq 0\} \tag{3}$$

contains the true system parameter β defined in (1), and for any $\gamma \in \mathscr{B}$ there exist constants $L(\gamma)>0$ and $M(\gamma)>0$, such that for all $||x_2|| \geqslant L(\gamma)$,

$$||g(x_1,x_2,x_3)|| \leqslant M(\gamma)(|\gamma^\tau g(x_1,x_2,x_3)|), \qquad \forall (x_1,x_3) \in \mathbb{R}^{p_1+q}.$$

(A4) $\{w_t, \mathscr{F}_t\}$ is a martingale difference sequence, where $\{\mathscr{F}_t\}$ is a non-decreasing sequence of sub-σ-algebras. Assume also that

$$\sup_t E[|w_{t+1}|^2|\mathscr{F}_t] < \infty \quad \text{a.s.}$$

and

$$\sum_{t=1}^n w_t^2 = O(n). \tag{4}$$

Obviously, (A1) and (A4) are standard conditions. (A2) simply says that $||g(x_1,x_2,x_3)||$ grows to ∞ as $||x_2|| \to \infty$, while (A3) requires that its growth rate is unchanged when it is multiplied by any γ defined in Eq. (3).

We now give two examples to illustrate (A2) and (A3).

Example 1. Consider the following system with the multiplicative nonlinearity being a polynomial of y_t:

$$y_{t+1} = a_1 f_1(y_t, u_{t-1}) + \cdots + a_m f_m(y_t, u_{t-1}) + (b_0 + b_1 y_t + \cdots + b_l y_t^l)u_t + w_{t+1},$$

where $\alpha=(a_1,\ldots,a_m)^\tau$ and $\beta=(b_0,\ldots,b_l)^\tau$ are unknown parameters; $|f_i(x)| \leqslant M(||x||+1), x \in \mathbb{R}^2, 1 \leqslant i \leqslant m$, and $b_l \neq 0$, $l \geqslant 1$. Set $p_1=0$, $p_2=1$ and $q=1$, and define $g(x_1,x_2,x_3)=(1,x_2,\ldots,x_2^l)^\tau$, and $P(\gamma)=\gamma_l$ for $\gamma=(\gamma_0,\ldots,\gamma_l)^\tau \in \mathbb{R}^{l+1}$, then it is easy to see that (A2) and (A3) hold.

It is worth noting that the power l in the above example can be arbitrarily large. We next present an example where the multiplicative part also contains the input sequence u_t.

Example 2. Consider the system:

$$y_{t+1} = a_1 f_1(y_t, y_{t-1}, u_{t-1}) + \cdots + a_m f_m(y_t, y_{t-1}, u_{t-1}) + [b_0 + b_1 B_1(y_t) + b_2|y_{t-1}|^\delta + b_3 B_2(u_{t-1})]u_t + w_{t+1},$$

where, $\alpha=(a_1,\ldots,a_m)^\tau$, $\beta=(b_0,\ldots,b_3)^\tau$ are unknown parameters; $\delta>0$; $B_1(\cdot)$ and $B_2(\cdot)$ are two bounded functions; $|f_i(x)| \leqslant M(||x||+1), x \in R^3, 1 \leqslant i \leqslant m$, and $b_2 \neq 0$. To verify (A2) and (A3), we just set $p_1=p_2=q=1$, $g(x_1,x_2,x_3)=[1, B_1(x_1), |x_2|^\delta, B_2(x_3)]^\tau$, and $P(\gamma)=\gamma_2$ for $\gamma=(\gamma_0,\gamma_1,\gamma_2,\gamma_3)^\tau$.

Now, we consider the following weighted one-step-ahead control performance:

$$J(u_t) = E\{y_{t+1}^2 + \lambda u_t^2|\mathscr{F}_t\}, \quad \lambda>0. \tag{5}$$

Here, to guarantee the finiteness of the control energy, we do not choose the pure minimum variance cost

$J_1(u_t)=E\{y_{t+1}^2|\mathscr{F}_t\}$, since even for simple bilinear systems the usual minimum phase condition may not be satisfied (cf. [3]).

The optimal nonadaptive control law that minimizes Eq. (5) is given by

$$u_t=-\frac{[\alpha^\tau f(\varphi_t)][\beta^\tau g(\varphi_t)]}{[\beta^\tau g(\varphi_t)]^2+\lambda}. \tag{6}$$

For estimating the unknown parameters in this control law, we adopt the random regularization method introduced in [5] and the weighted least squares (WLS) algorithm proposed in [2] and further studied in [1, 5].

Set

$$\theta=[\alpha^\tau,\beta^\tau]^\tau, \tag{7}$$

$$\psi_t=[f^\tau(\varphi_t),g^\tau(\varphi_t)u_t]^\tau. \tag{8}$$

Let θ_t be the estimated values of θ, which are recursively defined by the following WLS algorithm:

$$\theta_{t+1}=\theta_t+a_tP_t\psi_t(y_{t+1}-\psi_t^\tau\theta_t), \tag{9}$$

$$P_{t+1}=P_t-a_tP_t\psi_t\psi_t^\tau P_t, \tag{10}$$

$$a_t=(\lambda_t^{-1}+\psi_t^\tau P_t\psi_t)^{-1}, \tag{11}$$

where the initial values θ_0 and $P_0>0$ can be chosen arbitrarily, $\{\lambda_t\}$ is the weighting sequence defined by

$$\lambda_t=\frac{1}{h(r_t)},\quad r_t=||P_0^{-1}||+\sum_{i=0}^{t}||\psi_i||^2 \tag{12}$$

with $h(x)=\log^{1+\delta}x$ ($\delta>0$), or see [5] for more general choices.

Since the estimate for β given by the above WLS may not belong to the set $\mathscr{B}$ defined by Eq. (3), we now resort to the random regularization method introduced in [5] to secure this.

Let $\{\zeta_t\}$ be an independent sequence of $(m+l)$-dimensional random vectors which are uniformly distributed on the unit ball $\{x\in\mathbb{R}^{m+l}:\ ||x||\leqslant 1\}$ and independent of $\{w_t\}$. Define $T_t(x)=|P(\theta_t+P_t^{1/2}x)|$, $x\triangleq(x_1\cdots x_{m+l})\in\mathbb{R}^{m+l}$, where $P(x_1\cdots x_{m+l})\triangleq P(x_{m+1}\cdots x_{m+l})$ is the polynomial function defined in Eq. (3). Take a number $\sigma\in(0,\sqrt{2}-1)$, and define a sequence $\{\eta_t\}$ recursively as follows:

$$\eta_t=\begin{cases}\zeta_t & \text{if } T_t(\zeta_t)\geqslant(1+\sigma)T_t(\eta_{t-1}),\\ \eta_{t-1} & \text{otherwise,}\end{cases} \tag{13}$$

with initial value $\eta_0=\zeta_0$. Let

$$\widehat{\theta}_t=\theta_t+P_t^{1/2}\eta_t, \tag{14}$$

by which we replace θ in Eq. (6), and get the following certainty-equivalence control:

$$u_t=-\frac{[\widehat{\alpha}_t^\tau f(\varphi_t)][\widehat{\beta}_t^\tau g(\varphi_t)]}{[\widehat{\beta}_t^\tau g(\varphi_t)]^2+\lambda},\quad \widehat{\theta}_t=[\widehat{\alpha}_t^\tau,\widehat{\beta}_t^\tau]^\tau. \tag{15}$$

Now, we state the main result of this paper.

Theorem 1. *For the system* (1), *let the conditions* (A1)–(A4) *be satisfied, and let the adaptive control law be defined by* Eqs. (8)–(15), *then the closed-loop system is globally stable, i.e., for any initial condition,*

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(y_t^2+u_t^2)<\infty\quad a.s.$$

3. Proof of Theorem 1

For simplicity of presentation, we introduce the following notation:

$$Y_t\triangleq(y_t,\dots,y_{t-p_1+1})^\tau\in\mathbb{R}^{p_1},$$

$$Y_t'\triangleq(y_t,\dots,y_{t-p_2+1})^\tau\in\mathbb{R}^{p_2},$$

$$U_t\triangleq(u_t,\dots,u_{t-q+1})^\tau\in\mathbb{R}^{q}.$$

Then

$$\varphi_t=\begin{pmatrix}Y_t\\ Y_{t-p_1}'\\ U_{t-1}\end{pmatrix},$$

and we have

$$f(\varphi_t)=f(Y_t,Y_{t-p_1}',U_{t-1}),$$

$$g(\varphi_t)=g(Y_t,Y_{t-p_1}',U_{t-1}).$$

Remark 1. By Condition (A2), we know that the boundedness of $g(Y_t,Y_{t-p_1}',U_{t-1})$ depends mainly on Y_{t-p_1}', so from now on we shall write $g(Y_t,Y_{t-p_1}',U_{t-1})$ as $g(Y_{t-p_1}')$ for simplicity. This should cause no confusion in the analysis.

Following the proof of Theorem 2 in [5], we have the following basic result.

Lemma 1. *Let Conditions* (A3) *and* (A4) *be satisfied. Then for the parameter estimate $\widehat{\theta}_t$ defined by* Eqs. (8)–(14), *we have*

(i) $\lim_{t\to\infty}\widehat{\theta}_t=\widehat{\theta}_\infty$ *a.s.*,

(ii) $\sum_{t=1}^{n}(\psi_t^\tau\widetilde{\theta}_t)^2=\mathrm{o}(r_n)+\mathrm{O}(1)$ *a.s.*,

(iii) $\widehat{\beta}_\infty\in\mathscr{B}$, *where* $\widehat{\theta}_\infty = (\widehat{\alpha}_\infty^\tau \widehat{\beta}_\infty^\tau)^\tau$ *is a random vector and* $\widetilde{\theta}_t \triangleq \theta - \widehat{\theta}_t$; $\psi_t, r_t, \mathscr{B}$ *are defined by* Eqs. (8), (12) *and* (3), *respectively*.

Lemma 2. *Under conditions* (A1)–(A4), *the prediction errors are dominated by the input/output signals in the sense that*

$$\sum_{t=1}^{n} |\psi_t^\tau \widetilde{\theta}_t|^2 = \mathrm{o}\left(\sum_{t=1}^{n}(y_{t+1}^2 + u_t^2)\right) + \mathrm{o}(n).$$

Proof. By Eqs. (1) and (8) and Conditions (A1)–(A3), we have

$$\begin{aligned}
||\psi_t||^2 &= ||f(\varphi_t)||^2 + ||g(\varphi_t)u_t||^2 \\
&= ||f(\varphi_t)||^2 + \{||g(\varphi_t)u_t||^2\}\{I(||Y'_{t-p_1}|| \\
&\qquad \geqslant L(\beta)) + I(||Y'_{t-p_1}|| < L(\beta))\} \\
&\leqslant \mathrm{O}(||\varphi_t||^2) + \mathrm{O}(|\beta^\tau g(\varphi_t)u_t|^2) + \mathrm{O}(u_t^2) \\
&= \mathrm{O}(||\varphi_t||^2) + \mathrm{O}(|y_{t+1} - \alpha^\tau f(\varphi_t) - w_{t+1}|^2) \\
&\quad + \mathrm{O}(u_t^2) \\
&\leqslant \mathrm{O}(|y_{t+1}|^2 + ||\varphi_t||^2 + w_{t+1}^2) + \mathrm{O}(u_t^2),
\end{aligned}$$

which combined with (ii) in Lemma 1 and Eq. (4) yields the desired result. □

Lemma 3. *For* $x = (x_1, x_2, x_3) \in \mathbb{R}^{p_1+p_2+q}$, *there exists some constant* $T > 0$, *such that uniformly for* $t > T$ *and* $(x_1, x_3) \in \mathbb{R}^{p_1+q}$,

$$|\widehat{\beta}_t^\tau g(x)| \to \infty \quad \text{as } ||x_2|| \to \infty.$$

Proof. By Condition (A3) and (iii) in Lemma 1, we know that there exist $M_1 > 0$ and $M_2 > 0$, such that on $\{||x_2|| > M_1\}$,

$$||g(x)|| \leqslant M_2 |\widehat{\beta}_\infty^\tau g(x)|. \tag{16}$$

From this and (i) in Lemma 1, it follows that on $\{||x_2|| > M_1\}$,

$$\begin{aligned}
|\widehat{\beta}_\infty^\tau g(x)| &\leqslant |\widehat{\beta}_t^\tau g(x)| + |(\widehat{\beta}_\infty^\tau - \widehat{\beta}_t^\tau) g(x)| \\
&= |\widehat{\beta}_t^\tau g(x)| + \mathrm{o}(||g(x)||) \\
&= |\widehat{\beta}_t^\tau g(x)| + \mathrm{o}(|\widehat{\beta}_\infty^\tau g(x)|).
\end{aligned}$$

Therefore, there exists some constant $T > 0$ such that when $t > T$ and $||x_2|| > M_1$,

$$||\widehat{\beta}_\infty^\tau g(x)|| \leqslant 2||\widehat{\beta}_t^\tau g(x)||. \tag{17}$$

Consequently, the lemma follows easily from Eqs. (16), (17) and (A2). □

Lemma 4. *For any* $\varepsilon > 0$, *there exists* $M > 0$, *such that whenever* $|y_{t+1}| \leqslant M$ *we have*

$$\begin{aligned}
|u_t| &\leqslant \mathrm{O}(|\psi_t^\tau \widetilde{\theta}_t| + |w_{t+1}|) + \varepsilon(||Y_t|| + ||Y'_{t-p_1}|| \\
&\qquad + ||U_{t-1}||) + \mathrm{O}(1).
\end{aligned}$$

Proof. By Remark 1, we rewrite Eq. (15) as

$$u_t = -\frac{\widehat{\alpha}_t^\tau f(Y_t, Y'_{t-p_1}, U_{t-1}) \cdot \widehat{\beta}_t^\tau g(Y'_{t-p_1})}{[\widehat{\beta}_t^\tau g(Y'_{t-p_1})]^2 + \lambda}, \tag{18}$$

then it can be readily verified that

$$\psi_t^\tau \widehat{\theta}_t = \frac{\lambda \widehat{\alpha}_t^\tau f(Y_t, Y'_{t-p_1}, U_{t-1})}{[\widehat{\beta}_t^\tau g(Y'_{t-p_1})]^2 + \lambda} \tag{19}$$

and

$$y_{t+1} = \psi_t^\tau \widetilde{\theta}_t + \psi_t^\tau \widehat{\theta}_t + w_{t+1}. \tag{20}$$

By Eq. (18), the linear growth condition (A1) and boundedness of the estimates we have

$$|u_t| \leqslant \frac{|\widehat{\alpha}_t^\tau f(Y_t, Y'_{t-p_1}, U_{t-1})|}{\sqrt{[\widehat{\beta}_t^\tau g(Y'_{t-p_1})]^2 + \lambda}} \tag{21}$$

$$= \mathrm{O}\left(\frac{||Y_t|| + ||Y'_{t-p_1}|| + ||U_{t-1}|| + 1}{\sqrt{[\widehat{\beta}_t^\tau g(Y'_{t-p_1})]^2 + \lambda}}\right). \tag{22}$$

Next, By Eq. (22) and Lemma 3, we know that for any $\varepsilon > 0$, there exits $M > 0$, such that whenever $||Y'_{t-p_1}|| > M$ we have

$$|u_t| \leqslant \mathrm{O}(1) + \varepsilon(||Y_t|| + ||Y'_{t-p_1}|| + ||U_{t-1}||). \tag{23}$$

We can choose a large M to make $\varepsilon > 0$ small enough for later analysis.

Next, combining Eqs. (18)–(20), we get

$$\begin{aligned}
-\lambda u_t &= \psi_t^\tau \widehat{\theta}_t \cdot \widehat{\beta}_t^\tau g(Y'_{t-p_1}) \\
&= y_{t+1} \cdot \widehat{\beta}_t^\tau g(Y'_{t-p_1}) \\
&\quad - (\psi_t^\tau \widetilde{\theta}_t + w_{t+1}) \cdot \widehat{\beta}_t^\tau g(Y'_{t-p_1}).
\end{aligned} \tag{24}$$

Now, if $||Y'_{t-p_1}|| > M$, then by Eq. (23) the lemma is true. Otherwise, by Eq. (24) and the assumption $|y_{t+1}| \leqslant M$ we have

$$|u_t| \leqslant \mathrm{O}(|\psi_t^\tau \widetilde{\theta}_t| + |w_{t+1}|) + \mathrm{O}(1)$$

and hence the lemma is also true.

Proof of Theorem 1. By Eq. (22) and Lemma 3 we have

$$\begin{aligned}||U_t|| &\leqslant |u_t| + ||U_{t-1}|| \\ &\leqslant \mathrm{O}(||Y_t|| + ||U_{t-1}|| + 1) + \mathrm{o}(||Y'_{t-p_1}||). \quad (25)\end{aligned}$$

In a similar way, by Eq. (19) we have

$$\begin{aligned}|\psi_t^\tau \widehat{\theta}_t| &\leqslant \mathrm{O}\left(\frac{||Y_t|| + ||Y'_{t-p_1}|| + ||U_{t-1}|| + 1}{[\widehat{\beta}_t^\tau g(Y'_{t-p_1})]^2 + \lambda}\right) \quad (26) \\ &\leqslant \mathrm{O}\,(||Y_t|| + ||U_{t-1}|| + 1) + \mathrm{o}(||Y'_{t-p_1}||). \quad (27)\end{aligned}$$

Consequently, by Eq. (20) we have

$$\begin{aligned}||Y_{t+1}|| &\leqslant ||Y_t|| + |y_{t+1}| \\ &\leqslant ||Y_t|| + |\psi_t^\tau \widetilde{\theta}_t| + |\psi_t^\tau \widehat{\theta}_t| + |w_{t+1}| \quad (28) \\ &\leqslant \mathrm{O}(||Y_t|| + ||U_{t-1}|| + 1) \\ &\quad + \mathrm{o}(||Y'_{t-p_1}||) + |\psi_t^\tau \widetilde{\theta}_t| + |w_{t+1}|. \quad (29)\end{aligned}$$

Combining Eq. (25) with Eq. (29) we obtain

$$\begin{aligned}||Y_{t+1}|| + ||U_t|| &\leqslant \mathrm{O}(||Y_t|| + ||U_{t-1}||) + \mathrm{O}(1) \\ &\quad + \mathrm{o}(||Y'_{t-p_1}||) + |\psi_t^\tau \widetilde{\theta}_t| + |w_{t+1}|.\end{aligned}$$

Iterating this linear inequality backwards p_1 times and noting that $||Y_{t-p_1}|| \leqslant ||Y'_{t-p_1}||$ we get

$$\begin{aligned}||Y_t|| + ||U_{t-1}|| &\leqslant \mathrm{O}(||Y'_{t-p_1}|| + ||U_{t-p_1-1}||) + \mathrm{O}(1) \\ &\quad + \mathrm{o}\left(\sum_{i=1}^{p_1} ||Y'_{t-p_1-i}||\right) \\ &\quad + \mathrm{O}\left(\sum_{i=1}^{p_1} [|\psi_{t-i}^\tau \widetilde{\theta}_{t-i}| + |w_{t+1-i}|]\right). \quad (30)\end{aligned}$$

By Lemma 4 and the assumption $p_2 \geqslant q$, it is easy to see that if $||Y'_{t-p_1}|| \leqslant M$ then

$$\begin{aligned}||U_{t-p_1-1}|| &\leqslant \mathrm{O}\left(\sum_{i=p_1+1}^{p_1+q} [|\psi_{t-i}^\tau \widetilde{\theta}_{t-i}| + |w_{t+1-i}|]\right) \\ &\quad + \varepsilon \sum_{i=p_1+1}^{p_1+q} (||Y_{t-i}|| + ||Y'_{t-p_1-i}|| \\ &\quad + ||U_{t-1-i}||) + \mathrm{O}(1). \quad (31)\end{aligned}$$

Substituting this into Eq. (30) we have for $||Y'_{t-p_1}|| \leqslant M$

$$\begin{aligned}||Y_t|| + ||U_{t-1}|| &\leqslant \mathrm{O}\left(\sum_{i=1}^{p_1+q} [|\psi_{t-i}^\tau \widetilde{\theta}_{t-i}| + |w_{t+1-i}|]\right) \\ &\quad + \mathrm{O}(\varepsilon) \sum_{i=1}^{p_1+q} (||Y_{t-i}|| + ||Y'_{t-p_1-i}|| \\ &\quad + ||U_{t-1-i}||) + \mathrm{O}(1), \quad (32)\end{aligned}$$

which in turn substituted into Eq. (22) shows that for $||Y'_{t-p_1}|| \leqslant M$

$$\begin{aligned}|u_t| &\leqslant \mathrm{O}\left(\sum_{i=1}^{p_1+q} [|\psi_{t-i}^\tau \widetilde{\theta}_{t-i}| + |w_{t+1-i}|]\right) \\ &\quad + \mathrm{O}(\varepsilon) \sum_{i=1}^{p_1+q} (||Y_{t-i}|| + ||Y'_{t-p_1-i}|| \\ &\quad + ||U_{t-1-i}||) + \mathrm{O}(1). \quad (33)\end{aligned}$$

Combining this with Eq. (23) and noticing Lemma 2, it is easy to see that

$$\sum_{t=0}^{n} |u_t|^2 = \mathrm{O}(n) + \mathrm{O}(\varepsilon) \sum_{t=0}^{n} [|y_t|^2 + |u_{t-1}|^2]. \quad (34)$$

Similarly, substituting Eq. (32) into Eq. (26), and using Lemmas 2 and 3, we see that

$$\sum_{t=0}^{n} |\psi_t^\tau \widehat{\theta}_t|^2 = \mathrm{O}(n) + \mathrm{O}(\varepsilon) \sum_{t=0}^{n} [|y_t|^2 + |u_t|^2] \quad (35)$$

combining this with Eq. (20) and Lemma 2, we finally get

$$\sum_{t=0}^{n} |y_{t+1}|^2 = \mathrm{O}(n) + \mathrm{O}(\varepsilon) \sum_{t=0}^{n} [|y_t|^2 + |u_t|^2]. \quad (36)$$

Since, as can be easily checked, $\mathrm{O}(\varepsilon)$ can be made arbitrarily small, we get the desired stability result by combining Eqs. (34) and (36). □

4. Concluding remarks

In this paper, we have presented a stabilizing adaptive controller for a class of affine nonlinear stochastic systems. In contrast to the fundamental limitations on the additive nonlinearity found in [6,10], the nonlinear growth rate of the multiplicative part can be arbitrarily fast. This lends new insights into the adaptive stabilization of discrete-time nonlinear stochastic systems.

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