An Achievable Rate for the Multiple Level Relay Channel

Liang-Liang Xie[2]
Institute of Systems Science
Chinese Academy of Sciences
Beijing, 100080, China
e-mail: xie@control.iss.ac.cn

P. R. Kumar[1]
University of Illinois
CSL, 1308 West Main St.
Urbana, IL 61801-2307, USA
e-mail: prkumar@uiuc.edu

Consider a channel with $M+1$ nodes. Let the source node be denoted by 0, the destination node by M, and let the other $M-1$ nodes be denoted sequentially as $1, 2, \ldots, M-1$ in arbitrary order. Assume each Node $i \in \{0, 1, \ldots, M-1\}$ sends $x_i(t) \in \mathcal{X}_i$ at time t, and each Node $k \in \{1, 2, \ldots, M\}$ receives $y_k(t) \in \mathcal{Y}_k$ at time t, where the finite sets $\mathcal{X}_i$ and $\mathcal{Y}_k$ are the corresponding input and output alphabets. The channel dynamics is described by $p(y_1, y_2, \ldots, y_M | x_0, x_1, \ldots, x_{M-1})$. Finally, a one-step time delay is assumed at every relay node $i \in \{1, 2, \ldots, M-1\}$: $x_i(t) = f_{i,t}(y_i(t-1), y_i(t-2), \ldots)$, where $f_{i,t}$ can be designed to be any causal function.

The case of $M = 2$ (single relay) was considered in [1], where the following rate is proved to be achievable:

$$R < \max_{p(x_0,x_1)} \min\{I(X_0; Y_1|X_1),\, I(X_0, X_1; Y_2)\}. \qquad (1)$$

Later on, the coding scheme used for proving (1) was extended to the multiple level relay case ($M \geq 2$) in [2], and an achievable rate formula in a recursive constraint form was proved.

In [3], we proposed a new coding scheme and proved a new achievable rate formula for the Gaussian case. The scheme is simpler and avoids some inconvenient techniques (e.g., the Slepian-Wolf partitioning), although it coincides with [1] in giving the same achievable rate for the single relay case. More importantly, this new coding scheme is easier to extend to the multiple level relay case, and generally achieves higher rates than those proved in [2]. Here we present the results for the discrete memoryless case. A full version with additional discussions on feedback and degradedness can be found in [4].

The advantages of the new coding scheme of [3] have also been recognized by [5], where the corresponding achievable rate formula for the discrete memoryless case is also stated. The paper [5] also goes on to obtain the capacity of some relay channels under fading, which is the first significant capacity result for such channels, and one which may possibly constitute a breakthrough in the field.

Theorem 1 *For the discrete memoryless multiple level relay channel defined above, the following rate is achievable:*

$$R < \max_{p(x_0,x_1,\ldots,x_{M-1})} \min_{1\leq k\leq M} I(X_0, \ldots, X_{k-1}; Y_k | X_k, \ldots, X_{M-1}).$$

We provide a brief description of our coding scheme for the single relay case ($M = 2$) in the following. (For the Gaussian case, the coding scheme can be made even simpler as presented in [3].) Consider B blocks of transmission, each of T transmission slots. A sequence of $B-1$ indices, $w(b) \in \{1, \ldots, 2^{TR}\}, b = 1, 2, \ldots, B-1$ will be sent over in TB transmission slots.

Generation of codebooks Here one significant difference from [1] is that all the codebooks are of the same length 2^{TR}. (No more 2^{TR_0} as in [1]). Consider any fixed $p(x_0, x_1)$.

1) Generate at random 2^{TR} i.i.d. T-sequences in $\mathcal{X}_1^T$, each drawn according to $\text{Prob}(\mathbf{x}_1) = \prod_{t=1}^{T} p(x_{1,t})$. Index them as $\mathbf{x}_1(w_1)$, $w_1 \in \{1, 2, \ldots, 2^{TR}\}$.

2) For each $\mathbf{x}_1(w_1)$, generate 2^{TR} conditionally independent T-sequences $\mathbf{x}_0(w_0|w_1)$, $w_0 \in \{1, 2, \ldots, 2^{TR}\}$, drawn independently according to $\text{Prob}(\mathbf{x}_0|\mathbf{x}_1(w_1)) = \prod_{t=1}^{T} p(x_{0,t}|x_{1,t}(w_1))$. This defines the joint codebook for Nodes 0, 1:

$$\mathcal{C}_0 := \{\mathbf{x}_0(w_0|w_1),\, \mathbf{x}_1(w_1)\}. \qquad (2)$$

Repeating the above process 1)-2) *independently* once more, we generate another random codebook $\mathcal{C}_1$ similar to $\mathcal{C}_0$. We will use these two codebooks alternately as follows: In block $b = 1, \ldots, B$, the codebook $\mathcal{C}_{(b \bmod 2)}$ is used.

Encoding At the *beginning* of each block $b \in \{1, \ldots, B\}$, Node 1 has an estimate (see "Decoding" below) $\hat{w}_1(b-1)$ of $w(b-1)$, and sends $\mathbf{x}_1(\hat{w}_1(b-1))$ in the block, while Node 0 as the source sends $\mathbf{x}_0(w(b)|w(b-1))$. Let $\vec{Y}_k(b)$ denote the corresponding T-sequence received by Node $k \in \{1, 2\}$.

Decoding At the *end* of each block $b \in \{1, \ldots, B\}$, Node 1 declares that $\hat{w}_1(b) = w$ if w is the unique value in $\{1, \ldots, 2^{TR}\}$ such that in the block b,

$$\{\mathbf{x}_0(w|\hat{w}_1(b-1)),\, \mathbf{x}_1(\hat{w}_1(b-1)),\, \vec{Y}_1(b)\} \in A_\epsilon^{(T)}(X_0, X_1, Y_1).$$

Also, Node 2 declares that $\hat{w}_2(b-1) = w$ if w is the unique value in $\{1, \ldots, 2^{TR}\}$ such that in *both* the blocks b and $b-1$,

$$\{\mathbf{x}_1(w),\, \vec{Y}_2(b)\} \in A_\epsilon^{(T)}(X_1, Y_2), \quad \text{and}$$
$$\{\mathbf{x}_0(w|\hat{w}_2(b-2)),\, \mathbf{x}_1(\hat{w}_2(b-2)),\, \vec{Y}_2(b-1)\} \in A_\epsilon^{(T)}(X_0, X_1, Y_2).$$

Unlike the sequential manner in [1], our decoding is a *simultaneous* typicality check of the previous several blocks.

References

[1] T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, pp. 572–584, 1979.

[2] P. Gupta and P. R. Kumar, "Towards an information theory of large networks: an achievable rate region." *IEEE Trans. Inform. Theory*, pp.1877-1894, August 2003.

[3] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Trans. Inform. Theory*, May 2004.

[4] L.-L. Xie and P. R. Kumar, "An achievable rate for the multiple level relay channel," submitted to *IEEE Trans. Inform. Theory*, November 2003. http://decision.csl.uiuc.edu/∼prkumar

[5] G. Kramer, M. Gastpar, and P. Gupta, "Capacity theorems for wireless relay channels." *41st Allerton Conference on Comm., Contr., and Comp.*, October 2003. Monticello, Illinois.

[1]This material is based upon work partially supported by USARO under Contract Nos. DAAD19-00-1-0466 and DAAD19-01010-465, DARPA under Contract Nos. N00014-01-1-0576 and F33615-01-C-1905, AFOSR under Contract No. F49620-02-1-0217, DARPA/AFOSR under Contract No. F49620-02-1-0325, and NSF under Contract Nos. NSF ANI 02-21357 and CCR-0325716.

[2]This work was conducted visiting CSL.